TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Introduction to Automatic Structures

Exercise Sheet 5

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Notice

Exercise 20 b) is a challenging exercise. It will not be discussed during the tutorial. Solutions can be sent via e-mail to Felix Distel until the end of the year.

Exercise 18

The *Post Correspondence Problem (PCP)* is a well-known undecidable problem. The version we consider here is defined as follows. An *instance* $I = ((u_1, \dots, u_n), (v_1, \dots, v_n))$ *of PCP* consists of two sequences of words over the alphabet $\{0, 1\}$. We call $s = s_1 s_2 \cdots s_k \in \{1, \dots, n\}^+$ a *solution of* $((u_1, \dots, u_n), (v_1, \dots, v_n))$ iff

$$u_{s_1} u_{s_2} \cdots u_{s_k} = v_{s_1} v_{s_2} \cdots v_{s_k}.$$

The sequence $((\epsilon, \epsilon), (u_{s_1}, v_{s_1}), \dots, (u_{s_1} u_{s_2} \cdots u_{s_k}, v_{s_1} v_{s_2} \cdots v_{s_k}))$ is called the *construction of s*.

In this exercise we examine a connection between PCP and the FOL theory of the structure

$$\mathcal{A} = (\{0, 1, \#, \$\}^*; is_\epsilon, is_0, is_1, is_\#, is_\$, \circ),$$

where

- is_ϵ, is_0, is_1, $is_\#$ and $is_\$$ are unary predicates that decide whether a word is the empty word, the word 0, 1, # or \$, respectively, and
- the ternary relation $\circ$ checks whether in a given tuple (w_1, w_2, w_3) the word w_3 is the concatenation of w_1 and w_2.

a) Give FOL formulae that define the following relations in $\mathcal{A}$.
   - The prefix relation $\preceq$ and the suffix relation $\succeq$.
   - The substring relation $\subseteq$.
   - The unary relation $only_{01}$ that contains all words from $\{0, 1\}^*$.
   - For a given word $u \in \{0, 1, \#, \$\}^*$ the unary relation is_u contains only the word u itself.

b) For every instance $I = ((u_1, \dots, u_n), (v_1, \dots, v_n))$ of PCP define a FOL sentence ϕ_I with the following property: I has a solution iff ϕ_I belongs to the FOL theory of $\mathcal{A}$.

Hint: You can encode the construction of the solution s as the string

$$S = \#\$\#u_{s_1}\$v_{s_1}\#u_{s_1}u_{s_2}\$v_{s_1}v_{s_2}\#\cdots\#u_{s_1}u_{s_2}\cdots u_{s_k}\$v_{s_1}v_{s_2}\cdots v_{s_k}\#$$

c) Use your previous results to prove or refute the claim that $\mathcal{A}$ has an automatic presentation.

Exercise 19

Show that the following structures are isomorphic to structures that are FOL definable in the universal structure $(\{0,1\}^*; \preceq, S_0, S_1, \mathit{EqualLength})$. Give the corresponding FOL formulae.

a) $(\mathbb{N}; \leq)$

b) $(\mathbb{N}; +)$

Exercise 20

We consider universal structures again.

a) Give a structure $\mathcal{A}$ other than $(\{0,1\}^*; \preceq, S_0, S_1, \mathit{EqualLength})$ that is universal in the following sense: if the structure $\mathcal{B}$ has an automatic presentation then $\mathcal{B}$ has an automatic presentation that is FOL definable in $\mathcal{A}$.

b) Show that an automatic structure over the unary alphabet can never be universal.