TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Introduction to Automatic Structures

Exercise Sheet 6

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Exercise 21

As mentioned in the lecture, the equivalence problem for automatic equivalence relations is undecidable. Use this result to prove that the equivalence problem for trees of height ≤ 2 is undecidable.

Hint: Can you encode equivalence relations as trees of height 2?

Exercise 22

Show that the automatic isomorphism relation between automatic structures is an equivalence relation.

Exercise 23

Let Σ be an alphabet. For a language $L \subseteq \Sigma^*$ we define the language $L^\omega = \{\alpha \in \Sigma^\omega \mid \alpha = u_1 u_2 u_3 \cdots \text{ where } u_i \in L \setminus \{\epsilon\}\}$.

Let $L_1, L_2 \subseteq \Sigma^*$. Prove or refute:

a) $(L_1 \cup L_2)^\omega \subseteq L_1^\omega \cup L_2^\omega$

b) $(L_1 \cup L_2)^\omega \supseteq L_1^\omega \cup L_2^\omega$

Exercise 24

Give Büchi automata that recognize the following ω-regular languages over the alphabet $\Sigma := \{a, b, c\}$:

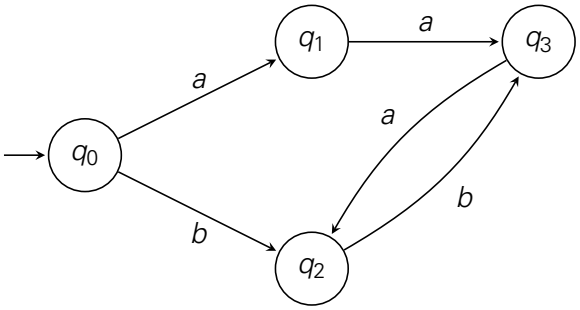
a) $\{\alpha \in \Sigma^\omega \mid$ the string *abc* occurs in $\alpha\}$

b) $\{\alpha \in \Sigma^\omega \mid$ the string *abc* occurs in α infinitely often$\}$

c) $(a^+b^+c^+)^\omega$, i.e. the language that consists of the pattern "finitely many *a*s, followed by finitely many *b*s, followed by finitely many *c*s" repeated infinitely ften.

Exercise 25

Consider Büchi automata using the following transition system:



Check whether the recognized language is empty for the following sets of final states.

a) $F = \{q_0, q_1\}$

b) $F = \{q_2, q_3\}$

c) $F = \{q_1, q_3\}$