Introduction to Automatic Structures

Exercise Sheet 6
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Exercise 21
As mentioned in the lecture, the equivalence problem for automatic equivalence relations is undecidable. Use this result to prove that the equivalence problem for trees of height \( \leq 2 \) is undecidable.

Hint: Can you encode equivalence relations as trees of height 2?

Exercise 22
Show that the automatic isomorphism relation between automatic structures is an equivalence relation.

Exercise 23
Let \( \Sigma \) be an alphabet. For a language \( L \subseteq \Sigma^* \) we define the language \( L^\omega = \{ \alpha \in \Sigma^\omega \mid \alpha = u_1 u_2 u_3 \cdots \text{ where } u_i \in L \setminus \{ \varepsilon \} \} \).
Let \( L_1, L_2 \subseteq \Sigma^* \). Prove or refute:

a) \((L_1 \cup L_2)^\omega \subseteq L_1^\omega \cup L_2^\omega\)

b) \((L_1 \cup L_2)^\omega \supseteq L_1^\omega \cup L_2^\omega\)

Exercise 24
Give Büchi automata that recognize the following \( \omega \)-regular languages over the alphabet \( \Sigma := \{a, b, c\} \):

a) \( \{ \alpha \in \Sigma^\omega \mid \text{the string } abc \text{ occurs in } \alpha \} \)

b) \( \{ \alpha \in \Sigma^\omega \mid \text{the string } abc \text{ occurs in } \alpha \text{ infinitely often} \} \)

c) \((a^+ b^+ c^+)^\omega\), i.e. the language that consists of the pattern “finitely many as, followed by finitely many bs, followed by finitely many cs” repeated infinitely often.
Exercise 25
Consider Büchi automata using the following transition system:

Check whether the recognized language is empty for the following sets of final states.

a) $F = \{q_0, q_1\}$
b) $F = \{q_2, q_3\}$
c) $F = \{q_1, q_3\}$