Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Introduction to Automatic Structures

Exercise Sheet 7

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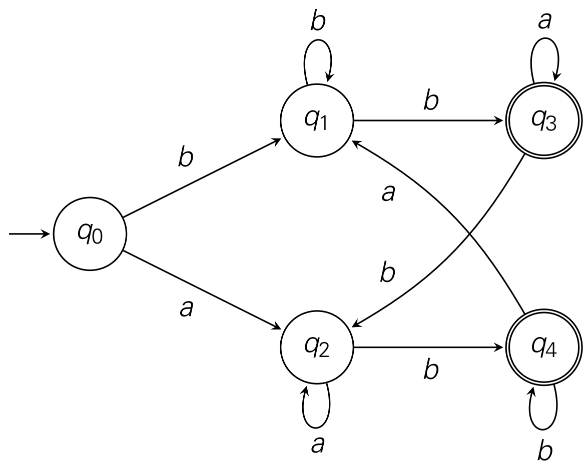
Exercise 26

Let $\Sigma = \{a, b\}$ and $L \subseteq \Sigma^\omega$ be the language recognized by the following Büchi-automaton:



Find a number $n \leq 1$ and regular languages $U_1, V_1, \ldots, U_n, V_n \subseteq \Sigma^*$ such that

$$\bigcup_{i=1}^{n} U_i \cdot V_i^\omega = L.$$

Exercise 27

Let Σ be an alphabet. Prove the following.

a) If $L \subseteq \Sigma^+$ is regular, then there exists a finite non-deterministic automaton $\mathcal{M}$ with only *one* final state such that $L = L(\mathcal{M})$.

b) If $L \subseteq \Sigma^*$ is regular, then there exists a finite non-deterministic automaton $\mathcal{M}$ with at most *two* final states such that $L = L(\mathcal{M})$.

c) There is *no* $k \geq 1$ such that the following holds: If $L \subseteq \Sigma^\omega$ is Büchi recognizable, then there exists a Büchi automaton $\mathcal{M}$ with at most k final states such that $L = L_\omega(\mathcal{M})$.

Hint: Consider the languages a^ω, $a^\omega \cup b^\omega$, $a^\omega \cup b^\omega \cup c^\omega$, ...

Exercise 28

Prove the claim from Theorem 5.7 from the lecture: Every word automata presentable structure is Büchi automata presentable.

Exercise 29

Give MSO-formulae that define the languages from Exercise 24:

a) $\{\alpha \in \Sigma^\omega \mid$ the string *abc* occurs in $\alpha\}$

b) $\{\alpha \in \Sigma^\omega \mid$ the string *abc* occurs in α infinitely often$\}$

c) $(a^+ b^+ c^+)^\omega$, i.e. the language that consists of the pattern "finitely many *a*s, followed by finitely many *b*s, followed by finitely many *c*s" repeated infinitely often.