Exercise 26
Let $\Sigma = \{a, b\}$ and $L \subseteq \Sigma^\omega$ be the language recognized by the following Büchi-automaton:

![Büchi Automaton Diagram]

Find a number $n \leq 1$ and regular languages $U_1, V_1, \ldots, U_n, V_n \subseteq \Sigma^*$ such that

$$\bigcup_{i=1}^{n} U_i \cdot V_i^\omega = L.$$

Exercise 27
Let $\Sigma$ be an alphabet. Prove the following.

a) If $L \subseteq \Sigma^+$ is regular, then there exists a finite non-deterministic automaton $M$ with only one final state such that $L = L(M)$.

b) If $L \subseteq \Sigma^*$ is regular, then there exists a finite non-deterministic automaton $M$ with at most two final states such that $L = L(M)$.

c) There is no $k \geq 1$ such that the following holds: If $L \subseteq \Sigma^\omega$ is Büchi recognizable, then there exists a Büchi automaton $M$ with at most $k$ final states such that $L = L_\omega(M)$.

Hint: Consider the languages $a^\omega, a^\omega \cup b^\omega, a^\omega \cup b^\omega \cup c^\omega$, \ldots
Exercise 28
Prove the claim from Theorem 5.7 from the lecture: Every word automata presentable structure is Büchi automata presentable.

Exercise 29
Give MSO-formulae that define the languages from Exercise 24:

a) \( \{ \alpha \in \Sigma^\omega \mid \text{the string } abc \text{ occurs in } \alpha \} \)

b) \( \{ \alpha \in \Sigma^\omega \mid \text{the string } abc \text{ occurs in } \alpha \text{ infinitely often} \} \)

c) \( (a^+ b^+ c^+)^\omega \), i.e. the language that consists of the pattern “finitely many as, followed by finitely many bs, followed by finitely many cs” repeated infinitely often.