



Introduction to Automatic Structures

Exercise Sheet 8

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Exercise 30

Prove that every Büchi automata presentable structure is tree automata presentable.

Exercise 31

Show that Rabin automata can accept the following sets of $\{0, 1\}$ -labelled trees.

- $\{(\mathcal{T}, \nu) \mid \nu(x) = 1 \text{ for only finitely many } x \in \mathcal{T}\}$,
- $\{(\mathcal{T}, \nu) \mid \text{each path has infinitely many nodes labelled } 1\}$,
- $\{(\mathcal{T}, \nu) \mid \forall x \in \mathcal{T}: \text{if } \nu(x) = 1 \text{ then the subtree rooted at } x \text{ is labelled by } 0\text{s only}\}$
- $\{(\mathcal{T}, \nu) \mid \exists x \in \mathcal{T}: \nu(x) = 1\}$

Exercise 32

The Bü are extraterrestrials living in the Chi-Galaxy on the planet Aut. They are immortal. A Bü-week consists of the days Om-day, Aton-day, Ra-day and Bin-day. The Bü-calendar simply counts the days since the planet Aut was created on an Om-day.

In order not to displease the goddess Om the Bü

- may not work until they are 4 days old,
- have to work 10 days after this,
- after their 14th day of life they retire and work only on Om-days.

The life of a Bü can be described by an infinite word $w \in \{1, Om, Aton, Ra, Bin\}^\omega$, as follows. The word begins with the birthday in unary coding using 1s, followed by the sequence of free weekdays. For example the string

$$\ell_b = 1111 \underbrace{Om Aton Ra Bin}_{\text{childhood of 4 days}} \underbrace{Ra Bin Aton Ra Bin \dots}_{\text{infinite retirement}}$$

describes the life of a Bü b who was born on the 4th day since Aut's creation.

- Sketch a Büchi automaton that accepts these words.
- By order of Aton, the god of love, two Bü b_1 and b_2 are allowed to get married only on dates that are a common multiple of their birthdays. Let m be a function mapping ℓ_{b_1} and ℓ_{b_2} to the word $w_{b_1, b_2} \in \{1, Aton\}^\omega$ that has *Aton* exactly on the positions corresponding to the dates when it is possible for b_1 and b_2 to get married. Prove or disprove: there is a Büchi-automaton that accepts m .