Term Rewriting Systems

Exercise Sheet 5

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Exercise 21

Let $\rho$ and $\tau$ be substitutions.

a) Describe the variable range $\text{VRan}(\rho \tau)$.

b) Under what conditions do we have $\rho \tau = \tau$?

c) Under what conditions do we have $\tau \tau = \tau$?

Exercise 22

Let $E$ be a set of identities with some $\ell \approx r \in E$ such that $\ell$ is a variable or $\text{Var}(r) \not\subseteq \text{Var}(\ell)$.
Prove that $\rightarrow_E$ is not terminating.

Exercise 23

Let $G_1 := \{ f^3(a) \approx a, f^6(a) \approx a \}$ and $G_2 := \{ f^4(a) \approx a, f^6(a) \approx a \}$. Consider the congruence closures of $G_1$ and $G_2$ and verify whether $f^2(a) \approx a$ holds in $G_1$ or $G_2$.

Exercise 24

Let $G$ be a set of ground identities and $\text{CC}(G)$ the congruence closure of $G$. Complete the proof of Lemma 4.8 of the lecture by proving that $\rightarrow_G \subseteq \text{CC}(G)$.

Exercise 25

Let $u, x, y,$ and $z$ be variables. Use the unification algorithm recalled in the lecture to solve the following two unification problems:

a) $S_1 := \{ f(h(x), g(x, u)) \approx z, f(z, g(f(y, y), z)) \}$

b) $S_2 := \{ h(x, g(x, y), y) \approx z, h(x, g(a, y), y), z \approx h(x, g(x, b), b) \}$

Exercise 26

From the unification algorithm recalled in the lecture, design a direct decision procedure for the matching problem.

Hint: “Direct” means that no constants are introduced in the right term. Instead, the rules are to be modified such that the new algorithm returns “the input terms do not match” or a matcher for the input terms as soon as possible.