TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Term Rewriting Systems

Exercise Sheet 5

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Exercise 21

Let ρ and τ be substitutions.

    a) Describe the variable range $\mathrm{VRan}(\rho\tau)$.

    b) Under what conditions do we have $\rho\tau = \tau$?

    c) Under what conditions do we have $\tau\tau = \tau$?

Exercise 22

Let E be a set of identities with some $\ell \approx r \in E$ such that ℓ is a variable or $\mathrm{Var}(r) \not\subseteq \mathrm{Var}(\ell)$. Prove that $\to_E$ is not terminating.

Exercise 23

Let $G_1 := \{f^3(a) \approx a, f^5(a) \approx a\}$ and $G_2 := \{f^4(a) \approx a, f^6(a) \approx a\}$. Consider the congruence closures of G_1 and G_2 and verify whether $f^2(a) \approx a$ holds in G_1 or G_2.

Exercise 24

Let G be a set of ground identities and $\mathrm{CC}(G)$ the congruence closure of G. Complete the proof of Lemma 4.8 of the lecture by proving that $\to_G \subseteq \mathrm{CC}(G)$.

Exercise 25

Let u, x, y, and z be variables. Use the unification algorithm recalled in the lecture to solve the following two unification problems:

    a) $S_1 := \{f(h(x), g(x, u)) =^? f(z, g(f(y, y), z))\}$

    b) $S_2 := \{h(x, g(x, y), y) =^? h(x, g(a, y), y),\ z =^? h(x, g(x, b), b)\}$

Exercise 26

From the unification algorithm recalled in the lecture, design a direct decision procedure for the matching problem.

Hint: "Direct" means that no constants are introduced in the right term. Instead, the rules are to be modified such that the new algorithm returns "the input terms do not match" or a matcher for the input terms as soon as possible.