TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Term Rewriting Systems

Exercise Sheet 6

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Exercise 27

Prove the undecidability of the *uniform* halting problem by a reduction of the halting problem.

Exercise 28

Why do we need $\overrightarrow{\cdot}$ in the proof of Thm. 5.7? More precisely, for a Turing machine $\mathcal{M}$, let

$$\Sigma'_{\mathcal{M}} := \{s_0, \ldots, s_n\} \cup \{q_0, \ldots, q_p\} \cup \{\overrightarrow{\ell}, \overleftarrow{r}\}$$

and let $R'_{\mathcal{M}}$ be a rewrite system obtained from $R_{\mathcal{M}}$ by replacing both $\overrightarrow{s_i}$ and $\overleftarrow{s_i}$ with s_i. Give an example of a terminating Turing machine $\mathcal{M}$ for which $R'_{\mathcal{M}}$ does not terminate.

Exercise 29

Prove that the following is *not* a decision procedure for the termination of a ground term rewriting system $R = \{\ell_i \to r_i \mid 1 \leq i \leq n\}$:

> Generate all reduction sequences starting with r_1.
> If one of these sequences yields a term that has r_1 as a subterm, then answer "non-terminating." Otherwise, continue with r_2, etc.

Exercise 30

Prove the "left version" of Thm. 5.9: Termination of a finite, left-ground term rewriting system is decidable.

Exercise 31

We define the order $>$ on terms as follows (recall that $|s|$ denotes the number of positions in s, and $|s|_x$ the number of occurrences of x in s):

$$s > t \text{ if } |s| > |t| \text{ and } |s|_x \geq |t|_x \text{ for all } x \in V.$$

Prove that $>$ is a reduction order on $\mathcal{T}(\Sigma, V)$.