TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Term Rewriting Systems

Exercise Sheet 7

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Exercise 32

The following problem is known as *Hilbert's 10th problem*, and it has been proved to be undecidable.
Given: One polynomial $P \in \mathbb{Z}[X_1, \ldots, X_n]$.
Question: Are there $a_1, \ldots, a_n \in \mathbb{N}$ such that $P(a_1, \ldots, a_n) = 0$?

Show that the undecidability of Hilbert's 10th problem implies the undecidability of the following problem, which you know from the lecture:
Given: Two polynomials $P, Q \in \mathbb{N}[X_1, \ldots, X_n]$ and a decidable set $A \subseteq \mathbb{N} \setminus \{0\}$.
Question: Is $P >_A Q$? That is, is $P(a_1, \ldots, a_n) > Q(a_1, \ldots, a_n)$ for all $a_1, \ldots, a_n \in A$?

Exercise 33

Let $P \in \mathbb{Z}[X]$ be a polynomial with one indeterminate and coefficients in $\mathbb{Z}$.

a) Prove for all $r \in \mathbb{Z}$: if $P(r) = 0$, then $r | a_0$, i.e. any root of P divides a_0.

b) Devise a decision procedure, which for each polynomial $P \in \mathbb{Z}[X]$ decides whether P has a root in $\mathbb{Z}$.

c) Show that for polynomials with more than one indeterminate, the roots need not satisfy such a property.

Exercise 34

Consider the TRS $R = \{g(x, g(y, z)) \to g(g(x, y), z),\ g(g(x, y), z) \to g(y, y)\}$. Use a polynomial interpretation to prove that R terminates.

Hint: Try $p_g = xy + y^2$.

Exercise 35

A TRS R is called *right-irreducible* if each r with $\ell \to r \in R$ is irreducible. Prove or refute the following claim: If R is right-ground and right-irreducible, then R is terminating.

Exercise 36

Prove Lemma 5.20 from the lecture: Let $\mathcal{A}$ be a monotone polynomial interpretation of Σ. Then $f^{\mathcal{A}}$ is a monotone function for each $f \in \Sigma$.