TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Term Rewriting Systems

Exercise Sheet 8

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Exercise 37

Let R be a finite TRS. We use $|t|_f$ to denote the number of occurrences of the symbol $f \in \Sigma$ in the term t. Prove the following claim: There is a positive integer k_R such that $s \to_R t$ implies $|t|_f \leq k_R(|s|_f + 1)$ for all terms s, t.

Exercise 38

Let R_{Ack} be the TRS from the lecture with

$$R_{\text{Ack}} = \{a(0, y) \to s(y), a(s(x), 0) \to a(x, s(0)), a(s(x), s(y)) \to a(x, a(s(x), y))\}.$$

Prove that there is no primitive recursive function that provides an upper bound for the length of reductions given the length of the first term. More precisely, prove that there is no primitive recursive function f such that $t_1 \to_{R_{\text{Ack}}} t_2 \to_{R_{\text{Ack}}} \cdots \to_{R_{\text{Ack}}} t_k$ implies $k \leq f(|t_1|)$.

Hint: Use the previous exercise and the fact the the Ackermann function grows faster than any primitive recursive function.

Exercise 39

For each of the following pairs (s, t) of terms, check whether $s \trianglerighteq_{\text{emb}} t$.

a) $f(x) \trianglerighteq_{\text{emb}} a$

b) $f(b) \trianglerighteq_{\text{emb}} a$

c) $g(g(x, y), g(a, f(z))) \trianglerighteq_{\text{emb}} g(y, g(a, z))$

Exercise 40

Prove the following claims:

a) The reduction relation $\xrightarrow{*}_{R_{\text{emb}}}$ given by the TRS

$$R_{\text{emb}} := \{f(x_1, \ldots, x_n) \to x_i \mid n \geq 1, f \in \Sigma^{(n)} \text{ and } 1 \leq i \leq n\}$$

and the homeomorphic embedding $\trianglerighteq_{\text{emb}}$ are identical, i.e. $s \xrightarrow{*}_{R_{\text{emb}}} t$ iff $s \trianglerighteq_{\text{emb}} t$.

b) $\trianglerighteq_{\text{emb}}$ is a partial order.

c) $\trianglerighteq_{\text{emb}}$ is well-founded. (Prove this without using Kruskal's Theorem.)

Exercise 41

In the proof of Thm. 5.32, we have used that $\trianglerighteq_{\text{emb}}$ is a well-partial-order. Explain why $\trianglerighteq_{\text{emb}}$ being a well-founded partial order would not have been sufficient.

Exercise 42

In lecture, it was shown that the termination of the TRS $R := \{f(f(x)) \to f(g(f(x)))\}$ cannot be proved using a simplification order.

a) Prove termination using the interpretation method.

b) Is there a polynomial order that can be used to prove termination of R?

Exercise 43

Prove that polynomial orders are simplification orders if the following properties are satisfied.

- The underlying signature Σ contains only function symbols of arity at least 2.
- The domain A does not contain 1, i.e. $A \subseteq \mathbb{N} \setminus \{0, 1\}$.

Are those conditions necessary?