TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Term Rewriting Systems

Exercise Sheet 9

Prof. Dr.-Ing. Franz Baader
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Exercise 44

Prove the first part of Thm. 5.38 of the lecture: Let Σ be a finite signature, $s, t \in \mathcal{T}(\Sigma, V)$, and $>_{\text{lpo}}$ be a lexicographic path order. We can decide whether $s >_{\text{lpo}} t$ in time polynomial in $|s|$ and $|t|$.

Hint: First, show that the condition

$$s >_{\text{lpo}} t_j \text{ for all } j \text{ with } 1 \leq j \leq n$$

in (LPO2c) can be replaced with

$$s >_{\text{lpo}} t_j \text{ for all } j \text{ with } i \leq j \leq n \text{ for } i \text{ such that } s_1 = t_1 \ldots s_{i-1} = t_{i-1}, \text{ and } s_i >_{\text{lpo}} t_i.$$

Use this modified condition to prove that the question whether $s >_{\text{lpo}} t$ holds can be decided in time $\mathcal{O}(|s| \cdot |t|)$.

Exercise 45

Prove that the termination of the TRS $R = \{f(f(x)) \to g(x), g(g(x)) \to f(x)\}$ cannot be proved using a lexicographic path order.

Exercise 46

Prove termination of the following TRS R using a lexicographic path order:

$$\begin{aligned} R = \{ & s(x) + (y + z) \to x + (s(s(y)) * z), \\ & s(x_1) + (x_2 + (x_3 + x_4)) \to x_1 + (x_2 + (x_3 + x_4))\} \end{aligned}$$

Exercise 47

Let Σ be finite signature with at least one constant symbol, $>$ a strict partial order on Σ, and $>_{\text{lpo}}$ the lexicographic path order induced by $>$. Prove the following claim: If $>$ is a total order on Σ, then $>_{\text{lpo}}$ is total on ground terms.

Exercise 48

Prove the following claim: If $>$ is a reduction order on $\mathcal{T}(\Sigma, V)$ that is total on ground terms, then $>$ satisfies the subterm property on ground terms, i.e. for each ground term t and position $p \in \text{Pos}(t) \setminus \{\varepsilon\}$, we have $t > t|_p$.