TECHNISCHE UNIVERSITÄT DRESDEN

**Faculty of Computer Science** Institute of Theoretical Computer Science, Chair of Automata Theory

# Term Rewriting Systems

## Exercise Sheet 10

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### Exercise 49

Complete the proof of Thm. 6.1 of the lecture: Let $E$ be a set of identities over $\Sigma$. Prove the following equivalence for all terms $s, t \in T(\Sigma, V)$:

$$s \approx_E t \text{ iff } \text{const}(s) \approx_E \text{const}(t),$$

where $\text{const}(\cdot)$ is a function that replaces every occurrence of a variable $x$ with a constant $a_x \notin \Sigma$.

### Exercise 50

Find terms $r_1, r_2$ such that $\{f(g(x)) \to r_1, g(h(x)) \to r_2\}$ is confluent.

### Exercise 51

Compute all critical pairs for the TRS consisting of the following rules:

$$0 + y \to y, \;\; s(x) + y \to s(x + y)$$
$$x + 0 \to x, \;\; x + s(y) \to s(x + y)$$

Is the system locally confluent? Is it convergent?

### Exercise 52

Finish Example 6.8 from the lecture: Show that the TRS $\{f(f(x)) \to g(x), f(g(x)) \to g(f(x))\}$ is terminating and confluent.

### Exercise 53

Consider the decision procedure in Corollary 6.7 for the confluence of finite, terminating TRS. Can you establish an upper bound for the runtime of the procedure as a function of the size of the input TRS?

### Exercise 54

Consider the system $\{f(x) \to g(x, y)\}$. Does it have any critical pairs? Is the induced rewrite relation confluent? What is going wrong here?

**Exercise 55**

Show that the TRS

$$\{(x * y) * (y * z) \to y,\ x * ((x * y) * z) \to x * y,\ (x * (y * z)) * z \to y * z\}$$

is confluent.