



## Term Rewriting Systems

### Exercise Sheet 11

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#### Exercise 56

Consider the following sets of identities:

$$E_1 := \{f(g(f(x))) \approx x\}$$

$$E_2 := \{f(g(f(x))) \approx f(g(x))\}$$

- Apply the basic completion procedure to  $E_1$  and  $E_2$ .
- What happens if the improved completion procedure that also simplifies rules is applied to  $E_1$ ?

#### Exercise 57

Show that the encompassment quasi-order  $\sqsupseteq$  is in fact a quasi-order and that the associated strict order  $\sqsubset$  is a well-founded strict order.

#### Exercise 58

Let  $\equiv$  denote the equivalence relation associated to  $\sqsupseteq$ , i.e.  $s \equiv t$  iff  $s \sqsupseteq t$  and  $t \sqsupseteq s$ . Show that:

- $s \equiv t$  iff  $s$  and  $t$  are equal up to variable renaming.
- For a given term  $s$ , there exist up to variable renaming only finitely many terms  $t_i$  such that  $s \sqsupseteq t_i$ .