Term Rewriting Systems

Exercise Sheet 12
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Exercise 59
a) Consider the following set of identities:

\[ E := \{ f(f(x, y), z) \approx f(x, f(y, z)), \ f(x, x) \approx x, \ f(f(x, y), x) \approx x \} \]

Apply the rules of the improved completion procedure to \( E \). Use a strategy that resembles the basic completion procedure, but simplifies rules as follows: upon adding new rules, simplify old ones by means of \( \text{L-SIMPLIFY-RULE} \) and \( \text{R-SIMPLIFY-RULE} \).

Consider the proof

\[ P := \langle f(x, f(y, f(y, x))), \ f(x, f(f(y, y), x)), \ f(x, f(y, x)), \ f(f(x, y), x), \ x \rangle. \]

Construct a rewrite proof \( P' \) in \( R_\omega \) with \( P \succ_c P' \) using the proof of Lemma 7.21.

b) Consider the following set of identities:

\[ E := \{ x + (y + z) \approx (x + y) + z, \ f(x) + f(y) \approx f(x + y) \} \]

Apply the completion procedure described above to input \( E \) and the polynomial order induced by

\[ P_f(X) = X + 1, \ P_+(X, Y) = XY^2. \]

Exercise 60
Consider the following completion procedure for ground term rewriting systems:

**Input:** \( G_0 \), a finite set of ground identities over \( \Sigma \), >, a reduction order that is total on the set of ground terms over \( \Sigma \).

**Procedure:** Apply the rules \( \text{L-SIMPLIFY-RULE} \), \( \text{DELETE} \), and \( \text{ORIENT} \), until no more rule is applicable.

**Output:** A ground term rewriting system.

Show that this procedure

a) always terminates,

b) is fair,

c) is correct, and

d) never fails.