![TU Dresden Logo] **TECHNISCHE UNIVERSITÄT DRESDEN**

Term Rewriting Systems

Exercise Sheet 12

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Exercise 59

a) Consider the following set of identities:

$$E := \{f(f(x, y), z) \approx f(x, f(y, z)), \; f(x, x) \approx x, \; f(f(x, y), x) \approx x\}$$

Apply the rules of the improved completion procedure to E. Use a strategy that resembles the basic completion procedure, but simplifies rules as follows: upon adding new rules, simplify old ones by means of L-SIMPLIFY-RULE and R-SIMPLIFY-RULE.

Consider the proof

$$P := \langle f(x, f(y, f(y, x))), \quad f(x, f(f(y, y), x)), \quad f(x, f(y, x)), \quad f(f(x, y), x), \quad x \rangle \, .$$

Construct a rewrite proof P' in R_ω with $P \succ_\mathcal{C} P'$ using the proof of Lemma 7.21.

b) Consider the following set of identities:

$$E := \{x + (y + z) \approx (x + y) + z, \; f(x) + f(y) \approx f(x + y)\}$$

Apply the completion procedure described above to input E and the polynomial order induced by

$$P_f(X) = X + 1, \; P_+(X, Y) = XY^2.$$

Exercise 60

Consider the following completion procedure for ground term rewriting systems:

Input: G_0, a finite set of ground identities over Σ, $>$, a reduction order that is total on the set of ground terms over Σ.

Procedure: Apply the rules L-SIMPLIFY-RULE, DELETE, and ORIENT, until no more rule is applicable.

Output: A ground term rewriting system.

Show that this procedure

a) always terminates,

b) is fair,

c) is correct, and

d) never fails.