Term Rewriting Systems

Exercise Sheet 13

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Exercise 61

The semi-decision procedure described in the proof of Thm. 7.22 of the lecture is rather inefficient: For the input $s \approx_E t$, all R_i-normal forms of s and t are computed in the ith iteration of the "Repeat"-loop. Show that the following modification of the procedure still yields a semi-decision procedure for the word problem:

- Begin with $s_0 := s$ and $t_0 := t$.
- After the ith "Repeat"-loop, compute *one arbitrary* R_i-normal form s_i of s_{i-1} and *one arbitrary* R_i-normal form t_i of t_{i-1}.
- Output "yes" ($s \approx_E t$) iff there exists an n such that $s_n = t_n$.

Hint: Show the following: (1) For all $n \geq 0$, it holds that $s \approx_E s_n$ and $t \approx_E t_n$; and (2) Since R_∞ is terminating, there exists an n such that $s_n = s_m$ and $t_n = t_m$ for all $m \geq n$.

Exercise 62

Let $f_1, \ldots, f_k \in K[X_1, \ldots, X_n]$. Prove that $\langle f_1, \ldots, f_k \rangle$ is the smallest ideal that contains $f_1, \ldots, f_k$.

Exercise 63

Let J be an ideal of $K[X_1, \ldots, X_n]$. Show that J contains the zero polynomial, and that $\equiv_J$ is a congruence relation on $K[X_1, \ldots, X_n]$.

Exercise 64

Show that the order $\succ$ on M_n defined by $X_1^{k_1} \cdots X_n^{k_n} \succ X_1^{\ell_1} \cdots X_n^{\ell_n}$ iff

- $\sum_{i=1}^n k_i > \sum_{i=1}^n \ell_i$, or
- $\sum_{i=1}^n k_i = \sum_{i=1}^n \ell_i$ and $(k_1, \ldots, k_n) >_{\text{lex}}^n (\ell_1, \ldots, \ell_n)$

is an admissible total order.