TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Description Logics

Exercise Sheet 2

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Exercise 5

Extend the mapping τ_x of $\mathcal{ALC}$-concept descriptions to first-order formulas given in the lecture to the description logic $\mathcal{ALCQ}$, which augments $\mathcal{ALC}$ with qualified number restrictions.

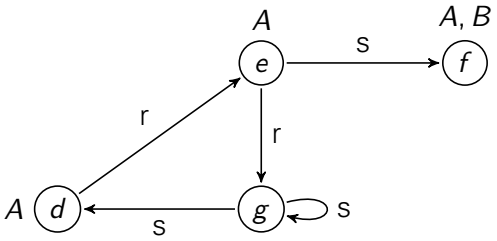
Exercise 6

Recall that the description logic $\mathcal{ALC}$ is equipped with the concept constructors negation ($\neg$), conjunction ($\sqcap$), disjunction ($\sqcup$), existential restriction ($\exists r.C$), and universal restriction ($\forall r.C$). Each subset of this set of constructors gives rise to a fragment of $\mathcal{ALC}$.

Identify all minimal fragments that are equivalent to $\mathcal{ALC}$ in the sense that for every $\mathcal{ALC}$-concept, there is an equivalent concept in the fragment. (Two concepts are equivalent iff they have the same extension in every interpretation.)

Exercise 7

Consider the (graphical representation of the) interpretation $\mathcal{I}$ with $\Delta^{\mathcal{I}} = \{d, e, f, g\}$:



For each of the following $\mathcal{ALCNI}$-concepts C, list all elements x of $\Delta^{\mathcal{I}}$ such that $x \in C^{\mathcal{I}}$:

a) $A \sqcup B$

b) $\exists s.\neg A$

c) $\forall s.A$

d) $(\geq 2\ s)$

e) $\exists s.\exists s.\exists s.\exists s.A$

f) $\forall s^{-1}.\exists s.\exists s.\exists s.A$

g) $\neg\exists r.(\neg A \sqcap \neg B)$

h) $\exists s.(A \sqcap \forall s.\neg B) \sqcap \neg\forall r.\exists r.(A \sqcup \neg A)$

Exercise 8

Consider the TBox

$$\mathcal{T} := \{\neg(A \sqcup B) \sqsubseteq \bot, \quad A \sqsubseteq \neg B \sqcap \exists r.B, \quad D \sqsubseteq \forall r.A, \quad B \sqsubseteq \neg A \sqcap \exists r.A\},$$

the ABox

$$\mathcal{A} := \{r(a,b), \;\; r(a,c), \;\; r(a,d), \;\; r(d,c), \;\; (B \sqcap \forall r.D)(a), \;\; E(b), \;\; (\neg A)(c), \;\; (\exists s.\neg D)(d)\},$$

and the knowledge base $\mathcal{K} := \langle \mathcal{T}, \mathcal{A} \rangle$. Check for

a) the TBox $\mathcal{T}$,

b) the ABox $\mathcal{A}$, and

c) the knowledge base $\mathcal{K}$

whether it has a model. If it has one, specify such a model. If it does not have a model, explain why.