Description Logics

Exercise Sheet 4

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Exercise 13

Prove the following results.

Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ be a knowledge base, C an $\mathcal{ALC}$-concept description, and a an individual name.

a) $\mathcal{K}$ is consistent iff $\tau(\mathcal{K})$ is consistent.

b) a is an instance of C w.r.t. $\mathcal{K}$ iff $\tau(\mathcal{K}) \models \tau_x(C)(a)$.

Exercise 14

In the lecture, we defined bisimulations for $\mathcal{ALC}$-concept descriptions s.t. they capture the expressive power of $\mathcal{ALC}$, i.e. that bisimulation invariance for $\mathcal{ALC}$-concept descriptions follows.

a) Extend the notion of bisimulation relation to $\mathcal{ALCN}$ s.t. bisimulation invariance for $\mathcal{ALCN}$-concept descriptions follows.

b) Show bisimulation invariance for the bisimulation relation defined in exercise (a).

c) Prove that $\mathcal{ALCQ}$ is more expressive than $\mathcal{ALCN}$.

Exercise 15

Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ be a consistent knowledge base. We write $C \sqsubseteq_{\mathcal{K}} D$ if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ for all models $\mathcal{I}$ of $\mathcal{K}$. Prove that for all $\mathcal{ALC}$-concepts C and D, we have $C \sqsubseteq_{\mathcal{K}} D$ iff $C \sqsubseteq_{\mathcal{T}} D$.

Hint: Use disjoint unions.

Exercise 16

Show the following claim:
If a concept C is satisfiable w.r.t. an $\mathcal{ALC}$-TBox $\mathcal{T}$, then for all $n \geq 1$ there is a model $\mathcal{I}_n$ of $\mathcal{T}$ such that: $|C^{\mathcal{I}_n}| \geq n$.