# Description Logics

## Exercise Sheet 5

Dr.-Ing. Anni-Yasmin Turhan
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### Exercise 17

Let $\rho_1 \subseteq \Delta^{\mathcal{I}_1} \times \Delta^{\mathcal{I}_2}$ and $\rho_2 \subseteq \Delta^{\mathcal{I}_2} \times \Delta^{\mathcal{I}_3}$ be bisimulation relations. Prove that bisimulations are closed under

a) composition (e.g. $\rho_3 = \rho_2 \circ \rho_1$ is a bisimulation), and

b) union (e.g. $\rho_3 = \rho_1 \cup \rho_2$ is a bisimulation).

### Exercise 18

Prove or refute the following claim:
Given an $\mathcal{ALC}$-concept $C$ and an $\mathcal{ALC}$-TBox $\mathcal{T}$. If $\mathcal{I}$ is an interpretation and $\mathcal{J}$ its filtration w.r.t. $\mathsf{Sub}(C) \cup \mathsf{Sub}(\mathcal{T})$, then the relation $\rho = \{(d, [d]) \mid d \in \Delta^{\mathcal{I}}, [d] \in \Delta^{\mathcal{J}},\ d \simeq [d]\}$ is a bisimulation.

### Exercise 19

Use the tableau algorithm from the lecture to decide whether the following subsumption holds:

$$\neg\forall r.A \sqcap \forall r.C \sqsubseteq_{\mathcal{T}} \forall r.E$$

where $\mathcal{T} = \{C \equiv (\exists r.\neg B) \sqcap \neg A, \quad D \equiv \exists r.B, \quad E \equiv \neg(\exists r.A) \sqcap \exists r.D\}$.