Fuzzy Logic

Solutions to Exercise Sheet 3

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Exercise 12

d) To show $BL \vdash \big((\varphi \to \psi) \wedge (\varphi \to \chi)\big) \to \big(\varphi \to (\psi \wedge \chi)\big)$ (8)

The idea is to show that both $BL \vdash (\psi \to \chi) \to (8)$ and $BL \vdash (\chi \to \psi) \to (8)$ and then use (A7), the axiom of case distinction.

We only prove $BL \vdash (\psi \to \chi) \to (8)$ in detail:

$BL \vdash (\varphi \to \psi) \to \big((\psi \to \psi \wedge \chi) \to (\varphi \to \psi \wedge \chi)\big)$	instance of (A1)
$BL \vdash \underbrace{(\varphi \to \psi) \,\&\, \big((\varphi \to \psi) \to (\varphi \to \chi)\big)}_{(\varphi\to\psi)\wedge(\varphi\to\chi)} \to (\varphi \to \psi)$	instance of (A2)
$BL \vdash (\varphi \to \psi) \wedge (\varphi \to \chi) \to \big((\psi \to \psi \wedge \chi) \to (\varphi \to \psi \wedge \chi)\big)$	transitivity, i.e. (A1) + 2× mod. pon.
$BL \vdash (\psi \to \psi \wedge \chi) \to \underbrace{\big((\varphi \to \psi) \wedge (\varphi \to \chi) \to (\varphi \to \psi \wedge \chi)\big)}_{(8)}$	(2) + mod. pon.
$BL \vdash (\psi \to \chi) \to (\psi \to \psi \wedge \chi)$	Exercise 12 c)
$BL \vdash (\psi \to \chi) \to (8)$	transitivity, i.e. (A1) + 2× mod. pon.

Using the same arguments one can show that

$$BL \vdash (\chi \to \psi) \to (8)$$

We can then use (A7) to prove the claim:

$BL \vdash \big((\psi \to \chi) \to (8)\big) \to \Big(\big((\chi \to \psi) \to (8)\big) \to (8)\Big)$	instance of (A7)
$BL \vdash \big((\chi \to \psi) \to (8)\big) \to (8)$	modus ponens
$BL \vdash (8)$	modus ponens

e) To show $\varphi \to \neg\neg\varphi$ (10)

$BL \vdash \big(\varphi \,\&(\varphi \to \mathbf{0})\big) \to \mathbf{0}$	instance of (4)
$BL \vdash \varphi \to \underbrace{\big((\varphi \to \mathbf{0}) \to \mathbf{0}\big)}_{\neg\neg\varphi}$	(A6) + mod. pon