TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Fuzzy Logic

Exercise Sheet 1

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Exercise 1

Show that the following three binary operators are continuous t-norms:

Łukasiewicz t-norm: $x \otimes y = \max\{x + y - 1, 0\}$,

Product t-norm: $x \otimes y = x \cdot y$,

Gödel t-norm: $x \otimes y = \min\{x, y\}$.

Exercise 2

A partial order on the set of all t-norms can be defined naturally as follows. Let $\otimes_1$ and $\otimes_2$ denote two t-norms. We write

$$\otimes_1 \leq \otimes_2 :\Leftrightarrow \forall u, v \in [0, 1] : u \otimes_1 v \leq u \otimes_2 v.$$

Find two t-norms $\otimes_{\min}$ and $\otimes_{\max}$ such that every t-norm $\otimes$ satisfies $\otimes_{\min} \leq \otimes \leq \otimes_{\max}$.

Exercise 3

Show that for every continuous t-norm and its residuum $\Rightarrow$, and every $x, y \in [0, 1]$

a) $x \leq y$ iff $x \Rightarrow y = 1$,

b) $(1 \Rightarrow x) = x$.

Exercise 4

Show that the following three binary operators are the residua of the t-norms from Exercise 1.

Łukasiewicz: $x \Rightarrow y = \begin{cases} 1 & \text{if } x \leq y \\ 1 - x + y & \text{otherwise} \end{cases}$

Product: $x \Rightarrow y = \begin{cases} 1 & \text{if } x \leq y \\ \frac{y}{x} & \text{otherwise} \end{cases}$

Gödel: $x \Rightarrow y = \begin{cases} 1 & \text{if } x \leq y \\ y & \text{otherwise} \end{cases}$