TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Fuzzy Logic

Exercise Sheet 2

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Exercise 5

Find the precomplement of the three continuous t-norms: Łukasiewicz t-norm, Product t-norm and Gödel t-norm.

Exercise 6

Using ordinal sums, construct a continuous t-norm where exactly 3 values from $[0, 1]$ are idempotent, i.e. exactly 3 values satisfy $x \otimes x = x$.

Exercise 7

Let the order $\leq$ on the set of all t-norms be defined as in Exercise 2:

$$\otimes_1 \leq \otimes_2 \iff \forall u, v \in [0, 1] \colon u \otimes_1 v \leq u \otimes_2 v?$$

Is there a continuous t-norm $\otimes_{\min}$ such that for every continuous t-norm $\otimes$ it holds that $\otimes_{\min} \leq \otimes$? Justify your answer by giving either an example or a proof.

Exercise 8

A t-norm $\otimes$ is called *nilpotent* if there are values $x, y \in (0, 1]$ such that $x \otimes y = 0$. Prove or disprove the following statements:

a) If $\otimes$ has idempotent elements in $(0, 1)$ then it is not nilpotent.

b) If $\otimes$ is isomorphic to the Łukasiewicz t-norm, then it is nilpotent.

c) If $\otimes$ is nilpotent, then it is not isomorphic to the product t-norm.

Exercise 9

Which of the following t-norms (if any) are isomorphic?

2nd Hamacher t-norm: $x \otimes_2^H y = \frac{xy}{xy-x-y+2}$

2nd Schweizer-Sklar t-norm: $x \otimes_2^{SS} y = \sqrt{\max\left\{x^2 + y^2 - 1, 0\right\}}$

2nd Yager t-norm: $x \otimes_2^Y y = \max\left\{1 - \sqrt{(1-x)^2 + (1-y)^2}, 0\right\}$