Fuzzy Logic

Exercise Sheet 3

Dr. Felix Distel
Winter Semester 2012

Exercise 10

Check for which of the three fundamental t-norms (if any) the following formulas are
1-tautologies.

  a) $\varphi \vee \neg\varphi$

  b) $\varphi \rightarrow \varphi \,\&\, \varphi$

  c) $(\varphi \wedge \psi) \rightarrow (\varphi \,\&\, \psi)$

Exercise 11

Prove that the axioms (A5)–(A8) are 1-tautologies of $PC(\otimes)$ for all continuous t-norms $\otimes$.

(A5) $\big(\varphi \rightarrow (\psi \rightarrow \chi)\big) \rightarrow (\varphi \,\&\, \psi \rightarrow \chi)$

(A6) $(\varphi \,\&\, \psi \rightarrow \chi) \rightarrow \big(\varphi \rightarrow (\psi \rightarrow \chi)\big)$

(A7) $\big((\varphi \rightarrow \psi) \rightarrow \chi\big) \rightarrow \Big(\big((\varphi \rightarrow \psi) \rightarrow \chi\big) \rightarrow \chi\Big)$

(A8) $\mathbf{0} \rightarrow \varphi$

Exercise 12

Using only modus ponens and the axioms (A1)–(A8) prove the following formulas in BL.

  a) $\varphi \rightarrow \varphi$ (3)

  b) $\big((\varphi \,\&\, \psi) \,\&\, \chi\big) \rightarrow \big(\varphi \,\&\, (\psi \,\&\, \chi)\big)$ (associativity of strong conjunction)

  c) $(\varphi \rightarrow \psi) \rightarrow \big(\varphi \rightarrow (\varphi \wedge \psi)\big)$

  d) $\big((\varphi \rightarrow \psi) \wedge (\varphi \rightarrow \chi)\big) \rightarrow \big(\varphi \rightarrow (\psi \wedge \chi)\big)$ (8)

  e) $\varphi \rightarrow \neg\neg\varphi$ (10)

Hint: to prove d) you can use c).

Exercise 13

Let $\varphi = x \rightarrow y$ and $\psi = x \, \& \, x \rightarrow y \, \& \, y$. Prove or disprove

  a) $\{\varphi\} \vdash \psi$

  b) $\text{BL} \vdash \varphi \rightarrow \psi$

  c) $\text{BL} \vdash \varphi \, \& \, \varphi \rightarrow \psi$