# Fuzzy Logic

## Exercise Sheet 4

Dr. Felix Distel
Winter Semester 2012

### Exercise 14

Let $(L, \leq)$ be a lattice where we denote the infimum (meet) by $\cap$ and the supremum (join) by $\cup$. Prove that the following properties hold for all $a, b \in L$:

a) $a \leq b$ iff $a \cap b = a$ iff $a \cup b = b$.

b) $a \cup (a \cap b) = a$ and $a \cap (a \cup b) = a$

### Exercise 15

Let $(L, \leq, \otimes, \Rightarrow, 1, 0)$ be a residuated lattice. Prove that the following statement holds for every $x, y, z \in L$.

$$(x \cup y) \otimes z = (x \otimes z) \cup (y \otimes z)$$

*Hint*: Show the two inequalities $(x \cup y) \otimes z \leq (x \otimes z) \cup (y \otimes z)$ and $(x \cup y) \otimes z \geq (x \otimes z) \cup (y \otimes z)$ separately.

### Exercise 16

Check whether the following structures are residuated lattices.

a) $\left\{\{0, 1\}^n, \leq, \wedge, \rightarrow, 0^n, 1^n\right\}$ for some fixed natural number $n$ where $\wedge$ and $\rightarrow$ denote the usual boolean operators taken pointwise and where we define

$$\mathbf{a} \leq \mathbf{b} \text{ iff } \mathbf{a} \rightarrow \mathbf{b} = 1^n$$

for all $\mathbf{a}, \mathbf{b} \in \{0, 1\}^n$.

b) $\{2^M, \supseteq, \Delta, /, M, \emptyset\}$ where $M$ is a finite set, $\supseteq$ and $\Delta$ denote the superset relation and symmetric difference, respectively, and $/$ is defined as $A \,/\, C := C \setminus A$.

### Exercise 17

Let $(L, \leq, \otimes, \Rightarrow, 0, 1)$ be a residuated lattice satisfying

- $\leq$ is a total order, and
- $x \cap y = x \otimes (x \Rightarrow y)$.

Then $(L, \leq, \otimes, \Rightarrow, 0, 1)$ is a BL-algebra.