TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Fuzzy Logic

Exercise Sheet 5

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Exercise 18

Prove Lemma 1.31, i.e. show that every divisible totally ordered residuated lattice is a BL-algebra.

Exercise 19

Show that the binary relation on $\mathcal{L}_T$ defined by

$$[\varphi]_T \leq [\psi]_T \text{ iff } T \vdash \varphi \to \psi$$

is well-defined and antisymmetric, i.e. prove that for all formulas φ, φ', ψ, ψ' it holds that

a) if $[\varphi]_T = [\varphi']_T$, $[\psi]_T = [\psi']_T$ and $T \vdash \varphi \to \psi$ then $T \vdash \varphi' \to \psi'$, and

b) if $[\varphi]_T \leq [\psi]_T$ and $[\psi]_T \leq [\varphi]_T$ then $[\varphi]_T = [\psi]_T$.

Exercise 20

Consider the residuated lattice from Exercise 16 a) for $n = 3$:

$$\{\{0,1\}^3, \leq, \wedge, \to, 000, 111\}$$

where $\wedge$ and $\to$ denote the usual boolean operators taken pointwise and where we define

$$\mathbf{a} \leq \mathbf{b} \text{ iff } \mathbf{a} \to \mathbf{b} = 111$$

for all $\mathbf{a}, \mathbf{b} \in \{0,1\}^3$.

a) How many filters does this residuated lattice have?

b) List the prime filters of this residuated lattice.

c) Give all the equivalence classes of $\sim_F$ for the filter $F = \{111, 011\}$.

Exercise 21

Let F_1 and F_2 be two filters on a linearly ordered BL-algebra $\mathbf{L}$. Prove or disprove the following properties.

a) $\{f_1 \otimes f_2 \mid f_1 \in F_1, f_2 \in F_2\}$ is a filter.

b) $\{f_1 \wedge f_2 \mid f_1 \in F_1, f_2 \in F_2\}$ is a filter.

c) $\{f_1 \vee f_2 \mid f_1 \in F_1, f_2 \in F_2\}$ is a filter.

Exercise 22

Let $\mathbf{L}$ be a countable BL-algebra and F a filter on $\mathbf{L}$. Let $z \in L$ be an element. Show that

$$F' := \{u \mid \exists v \in F \colon \exists n \in \mathbb{N} \colon v \otimes z^n \leq u\}$$

is the least filter containing F and z.