Fuzzy Logic

Exercise Sheet 6

Dr. Felix Distel
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Exercise 23

Prove the following properties for BL-algebras.

 a) $y \leq y \Rightarrow y^2$

 b) $x \Rightarrow y \leq x \otimes y \Rightarrow (y^2 \cup x^2)$

 c) $x \otimes y \leq x^2 \cup y^2$

 d) $(x \cup y)^2 = x^2 \cup y^2$

Exercise 24

Prove the following result: For every theory T, if $T \nvdash \varphi$ then there exists a consistent complete supertheory $T' \supseteq T$ such that $T' \nvdash \varphi$.

Exercise 25

Let **A** be an MV-algebra and T a theory. An **A**-evaluation $\mathcal{V}$ is called an **A**-*model* of T iff $\mathcal{V}$ evaluates each formula $\varphi \in T$ to 1.

By $[0,1]_{\text{Ł}}$ we denote the standard MV-algebra on $[0,1]$ defined by the truth functions of Łukasiewicz logic.

Define

$$T = \{np \to q \mid n \in \mathbb{N}\} \cup \{\neg p \to q\}$$

where $np = \neg(\neg p \& \neg(n-1)p)$. Prove that q is true in all $[0,1]_{\text{Ł}}$-models of T but this does not hold for any finite subset of T.

Exercise 26

Let $\otimes$ be a continuous t-norm and $\ominus$ its precomplement. Prove that the following are equivalent.

 a) $\ominus$ is involutive, i.e. $\ominus \ominus x = x$ for all $x \in [0,1]$, and

 b) $\otimes$ is equivalent to the Łukasiewicz t-norm.