Fuzzy Logic

Exercise Sheet 7

Dr. Felix Distel
Winter Semester 2012/13

Exercise 27

Show that in the product logic Π the axiom (Π2) can be equivalently replaced by any of the following formulae:

   a) $\neg(\varphi \mathbin{\&} \varphi) \to \neg\varphi$,

   b) $(\varphi \to \neg\varphi) \to \neg\varphi$, or

   c) $\neg\varphi \vee \neg\neg\varphi$.

Exercise 28

Show $\Pi \vdash (\varphi \mathbin{\&} \varphi \to \mathbf{0}) \to (\varphi \to \mathbf{0})$.

Exercise 29

Prove that the following sentences hold in every linearly ordered product algebra:

   a) if $x > 0$ then $\ominus x = 0$,

   b) if $z > 0$ then $x \otimes z = y \otimes z$ implies $x = y$, and

   c) if $z > 0$ then $x \otimes z < y \otimes z$ implies $x < y$.