

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

# **Fuzzy Logic**

#### **Exercise Sheet 7**

Dr. Felix Distel Winter Semester 2012/13

#### Exercise 27

Show that in the product logic  $\Pi$  the axiom ( $\Pi$ 2) can be equivalently replaced by any of the following formulae:

a) 
$$\neg(\varphi \& \varphi) \rightarrow \neg \varphi$$
,  
b)  $(\varphi \rightarrow \neg \varphi) \rightarrow \neg \varphi$ , or

c)  $\neg \varphi \lor \neg \neg \varphi$ .

### Exercise 28

Show  $\Pi \vdash (\varphi \& \varphi \to \mathbf{0}) \to (\varphi \to \mathbf{0}).$ 

## Exercise 29

Prove that the following sentences hold in every linearly ordered product algebra:

- a) if x > 0 then  $\ominus x = 0$ ,
- b) if z > 0 then  $x \otimes z = y \otimes z$  implies x = y, and
- c) if z > 0 then  $x \otimes z < y \otimes z$  implies x < y.