Theorems and problems

Theorem Proving with Equality

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1 Introduction

1. What is *saturation-based theorem proving*?
2. Difference between FOL and FOLE (which is more expressive and why)
3. Definition of a notion of *refutationally sound and refutationally complete* with respect to an inference system.

2 Binary Resolution $\mathcal{B}$

1. The inference system $\mathcal{B}$ is refutationally sound. (proof)
2. Orders: quasi-order, strict part of a quasi-order, reflexive closure of a strict order, well-founded order, Subterm Property (definitions)
3. Is a reflexive closure of a strict part of a quasi-order always identical with this quasi-order? (example)
4. The inference system $\mathcal{B}$ is refutationally complete for a set of ground clauses. (proof)
5. Let N be a set of ground clauses, C a ground clause, I_N, I_C interpretations constructed as in the definition of *model generation*. If $I_N \not\models C$ then $I_C \not\models C$.
6. The inference system $\mathcal{B}$ is refutationally complete. (proof)
7. Is the following set of clauses inconsistent:
   $\{P(z', z') \vee \neg Q(x), \neg P(a, y), P(x', b) \vee Q(f(x', x))\}$?

3 Ordered Binary Resolution $\mathcal{O}$

1. Justify refutational soundness of Ordered Binary Resolution.
2. Reduction order.
3. Reduction order on ground clauses has Subterm Property.
4. Refutational completeness of $\mathcal{O}$ for sets of ground clauses.(proof)
5. Lifting Lemma for Ordered Binary Resolution. (proof)

4 Other Versions of Resolution

1. Binary Resolution with Factoring (one rule)
2. Ordered Resolution with Selection
3. Ordered Resolution with Maximal Selection
4. Hyper-resolution (for sets of ground clauses)
5. Apply ordered resolution with maximal selection to
   $\{\neg S(x_1) \vee \neg P(x_1, b),\ \neg P(a, x_2) \vee Q(x_2, x_2),\ \neg Q(x_3, y) \vee R(x_3) \vee S(x_3)\}$.
6. Decide if the following propositions are true or false:
   - A set of clauses such that each of the clauses contains at least one positive literal is always satisfiable.
   - A set of clauses such that each of the clauses contains at least one negative literal is always satisfiable.

5 Redundancy

1. Redundancy Criterion (definition)
2. A set of clauses N is saturated up to redundancy with respect to an inference system Γ (definition)
3. Standard redundancy criterion (definition)
4. Standard redundancy criterion is a redundancy criterion (proof)
5. Given a set of ground clauses N such that N is saturated up to redundancy with respect to $\mathcal{B}$ (or $\mathcal{O}$) and the standard criterion $R^{\succ}$ N is unsatisfiable iff it contains a contradiction.
6. A clause C subsumes a clause D (definition -redundancy w.r.t. subsumption)
7. Deletion rules (4)
8. Fair derivation (definition)
9. If $D\sigma \in Gr(M)$, where M is a set of clauses, and $D\sigma$ is not redundant with respect to subsumption in $Gr(M)$, then D is not redundant with respect to subsumption in M. (proof)
10. Lifting Lemma for (Ordered) Binary Resolution with selection and redundancy deletion. (proof)
11. Satisfiability of a set of ground Horn clauses is decidable in linear time. (proof)

6 Superposition

1. Paramodulation, Ordered Paramodulation and Superposition (definitions)
2. Superposition inference system for ground Horn clauses $\mathcal{G}$.(definition)
3. The infenece system $\mathcal{G}$ is refutationally sound. (proof)
4. Equality Herbrand interpretation (definition)
5. The inference system $\mathcal{G}$ is refutationally complete for the set of ground Horn clauses. (proof)
6. The inference system $\mathcal{H}$ is refutationally complete for the set of ground Horn clauses. (proof)
7. Superposition inference system $\mathcal{I}$ (general case). (definition)
8. The inference system $\mathcal{I}$ is refutationally complete. (proof)