Assignment 3

Theorem Proving with Equality

Recall the definition of a lexicographic path order.

Let Σ be a finite signature and $>$ a strict order on Σ called *precedence*. The **lexicographic path order** $>_{lpo}$ on terms (or literals) induced by the precedence $>$, is defined as follows: $s >_{lpo} t$ iff

1. $t \in Var(s)$ and $s \neq t$,
2. $s = f(s_1, \ldots, s_m)$, $t = g(t_1, \ldots, t_n)$ and
   (a) there is i, $1 \leq i \leq m$, such that $s_i \geq_{lpo} t$ or
   (b) $f > g$ and $s >_{lpo} t_j$, for all j, $1 \leq j \leq n$ or
   (c) $f = g$, there is i, $1 \leq i \leq m$, such that $s_1 = t_1, \ldots, s_{i-1} = t_{i-1}$ and $s_i >_{lpo} t_i$ and $s >_{lpo} t_j$ for all j, $i < j \leq n$,

Problem 1. Let $\Sigma = \{f^{(2)}, i^{(1)}, e^{(0)}\}$ and $i > f > e$. Decide which is a bigger term with respect to $>_{lpo}$:

1. $f(x, e)$, x ?
2. $i(e)$, e?
3. $f(x, f(y, z))$, $f(f(x, y), z)$?

Problem 2. What should be done to use lpo as an admissible order on ground clauses?

Problem 3. Prove that if the precedence on the signature is total, then the lexicographic path order is total on the ground terms (or ground literals).

Hint: use induction on the size of both terms to be compared, (i.e. if $|s|$ and $|t|$ are sizes of the terms s and t, then the induction is on $|s| + |t|$).

Students may want to download Prover9 from:
http://www.cs.unm.edu/∼mccune/prover9/ to see some examples of proofs by resolution. (We will analyze some of the examples during the tutorial.)