Assignment 4

Theorem Proving with Equality

1. In the lecture we have seen an inference rule
   Binary resolution with factoring

   $$\frac{C \vee A_1 \vee \ldots \vee A_n \quad \neg B \vee D}{(C \vee D)\sigma}$$

   What is an ordered version of this rule?

2. Prove refutational soundness and completeness of:

   (a) **positive ordered resolution** (the selection function selects one and only one negative literal from a clause)

   (b) **ordered resolution with maximal selection**

3. A Horn clause is a clause with at most one positive literal. Prove refutational soundness and completeness of positive unit resolution for Horn clauses.

   Positive Unit Resolution:

   $$\frac{A \quad D \vee \neg B}{D\sigma}$$

   where $\sigma = mgu(A, B)$.

4. Apply ordered resolution with maximal selection to
   $\{\neg S(x_1) \vee \neg P(x_1, b),\ \neg P(a, x_2) \vee Q(x_2, x_2),\ \neg Q(x_3, y) \vee R(x_3) \vee S(x_3)\}$.

5. Decide if the following propositions are true or false:

   - A set of clauses such that each of the clauses contains at least one positive literal is always satisfiable.
   - A set of clauses such that each of the clauses contains at least one negative literal is always satisfiable.