Assignment 4
Theorem Proving with Equality

1. In the lecture we have seen an inference rule
   **Binary resolution with factoring**
   \[
   \frac{C \lor A_1 \lor \ldots \lor A_n \lor \neg B \lor D}{(C \lor D)\sigma}
   \]
   What is an ordered version of this rule?

2. Prove refutational soundness and completeness of:
   (a) **positive ordered resolution** (the selection function selects one and only one negative literal from a clause)
   (b) **ordered resolution with maximal selection**

3. A Horn clause is a clause with at most one positive literal. Prove refutational soundness and completeness of positive unit resolution for Horn clauses.
   **Positive Unit Resolution:**
   \[
   \frac{A \quad D \lor \neg B}{D\sigma}
   \]
   where \(\sigma = \text{mgu}(A, B)\).

4. Apply ordered resolution with maximal selection to
   \[
   \{-S(x_1) \lor \neg P(x_1, b), \neg P(a, x_2) \lor Q(x_2, x_2), \neg Q(x_3, y) \lor R(x_3) \lor S(x_3)\}.
   \]

5. Decide if the following propositions are true or false:
   - A set of clauses such that each of the clauses contains at least one positive literal is always satisfiable.
   - A set of clauses such that each of the clauses contains at least one negative literal is always satisfiable.