Chapter 4

Reasoning with Tableaux Algorithms

Outline

We start with an algorithm for deciding consistency of an ABox without a TBox.

This covers most of the inference problems introduced in Chapter 2:

- acyclic TBoxes can be eliminated by expansion
- satisfiability, subsumption, equivalence, and instance checking can be reduced to ABox consistency

Idea

The tableau-based consistency algorithm tries to generate a finite model for the input ABox $\mathcal{A}_0$

- tableau rules extend the ABox (one rule per constructor)
- obvious contradictions show inconsistency
- if an ABox is:

  complete (no rule applicable) and
  open (no contradictions found),

  then it describes a model

Example

$$\mathcal{T}: \quad \text{Heroine} \equiv \text{Hero} \sqcap \text{Female}$$

Subsumption:

$$\exists\text{helps.Hero} \sqcap \exists\text{helps.Female} \sqsubseteq^?_{\mathcal{T}} \exists\text{helps.Heroine}$$

Reduction to satisfiability: is the following concept unsatisfiable w.r.t. $\mathcal{T}$?

$$\exists\text{helps.Hero} \sqcap \exists\text{helps.Female} \sqcap \neg\exists\text{helps.Heroine}$$

Reduction to consistency: is the following ABox inconsistent w.r.t. $\mathcal{T}$?

$$\{(\exists\text{helps.Hero} \sqcap \exists\text{helps.Female} \sqcap \neg\exists\text{helps.Heroine})(a)\}$$

Expansion: is the following ABox inconsistent?

$$\{(\exists\text{helps.Hero} \sqcap \exists\text{helps.Female} \sqcap \neg\exists\text{helps.}(\text{Hero} \sqcap \text{Female}))(a)\}$$

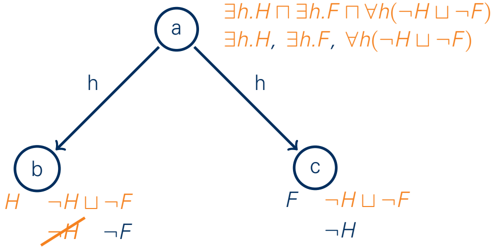
NNF: is the following ABox inconsistent?

$$\{(\exists\text{helps.Hero} \sqcap \exists\text{helps.Female} \sqcap \forall\text{helps.}(\neg\text{Hero} \sqcup \neg\text{Female}))(a)\}$$

Expansion of the ABox

Deciding inconsistency of

$$\{(\exists \text{helps.Hero} \sqcap \exists \text{helps.Female} \sqcap \forall \text{helps.}(\neg\text{Hero} \sqcup \neg\text{Female}))(a)\}$$



This is a complete, open ABox $\longrightarrow$ model of input ABox

It is consistent, subsumption does not hold

Formal Algorithm

Input An ABox $\mathcal{A}_0$

Output "yes" if $\mathcal{A}_0$ is consistent, "no" otherwise

Preprocessing: transform all concept descriptions of $\mathcal{A}_0$ to negation normal form

$$
\begin{array}{rcl}
\neg(C \sqcap D) & \rightsquigarrow & \neg C \sqcup \neg D \\
\neg(C \sqcup D) & \rightsquigarrow & \neg C \sqcap \neg D \\
\neg\neg C & \rightsquigarrow & C \\
\neg(\exists r.C) & \rightsquigarrow & \forall r.\neg C \\
\neg(\forall r.C) & \rightsquigarrow & \exists r.\neg C
\end{array}
$$

NNF transformation in polynomial time, is semantics invariant

Formal Algorithm (2)

Data Structure:
finite set of ABoxes. Initialized to $\{\mathcal{A}_0\}$ (in NNF)

Rule applications:
tableau rules replace one ABox from the set by finitely many new ABoxes

Termination:
when no rule can be applied to any ABox in the set

ABox is complete if no rule applies to it

Return:

"yes" if the set contains an open ABox
$\mathcal{A}$ is open if contains no obvious contradiction of the form $A(a), \neg A(a)$

"no" otherwise (i.e., if all ABoxes are closed)

Tableau Rules

There is one rule for each constructor (except negation)

$\sqcap$-rule

Condition: $\mathcal{A}$ contains $(C \sqcap D)(a)$ but not both $C(a)$ and $D(a)$

Action: $\mathcal{A}' := \mathcal{A} \cup \{C(a), D(a)\}$

$\sqcup$-rule

Condition: $\mathcal{A}$ contains $(C \sqcup D)(a)$ but neither $C(a)$ nor $D(a)$

Action: $\mathcal{A}' := \mathcal{A} \cup \{C(a)\}$ and $\mathcal{A}'' := \mathcal{A} \cup \{D(a)\}$

$\exists$-rule

Condition: $\mathcal{A}$ contains $(\exists r.C)(a)$ but there is no b with $\{r(a,b), C(b)\} \subseteq \mathcal{A}$

Action: $\mathcal{A}' := \mathcal{A} \cup \{r(a,b), C(b)\}$ where b is a new individual name

$\forall$-rule

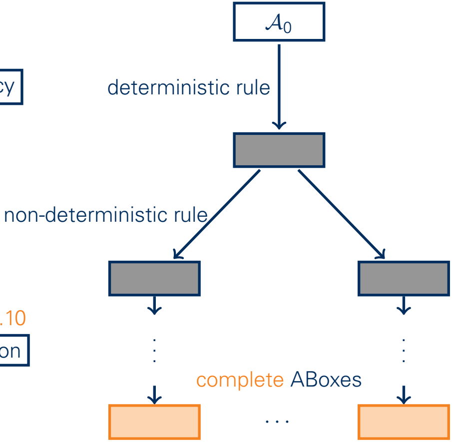
Condition: $\mathcal{A}$ contains $(\forall r.C)(a)$ and $r(a,b)$ but not $C(b)$

Action: $\mathcal{A}' := \mathcal{A} \cup \{C(b)\}$

Tableau Algorithm is a Decision Procedure

Lemma 4.1

rules preserve consistency



Lemma 4.10

termination

Lemma 4.4

soundness complete and open = consistent

completeness closed = inconsistent