



Description Logics

Exercise Sheet 2

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Exercise 5

Extend the mapping τ_x of \mathcal{ALC} -concept descriptions to first-order formulas given in the lecture to the description logic \mathcal{ALCQ} , which augments \mathcal{ALC} with qualified number restrictions.

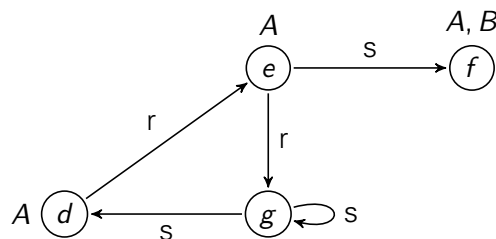
Exercise 6

Recall that the description logic \mathcal{ALC} is equipped with the concept constructors negation (\neg), conjunction (\sqcap), disjunction (\sqcup), existential restriction ($\exists r.C$), and universal restriction ($\forall r.C$). Each subset of this set of constructors gives rise to a fragment of \mathcal{ALC} .

Identify all minimal fragments that are equivalent to \mathcal{ALC} in the sense that for every \mathcal{ALC} -concept, there is an equivalent concept in the fragment. (Two concepts are equivalent iff they have the same extension in every interpretation.)

Exercise 7

Consider the (graphical representation of the) interpretation \mathcal{I} with $\Delta^{\mathcal{I}} = \{d, e, f, g\}$:



For each of the following \mathcal{ALCNI} -concepts C , list all elements x of $\Delta^{\mathcal{I}}$ such that $x \in C^{\mathcal{I}}$:

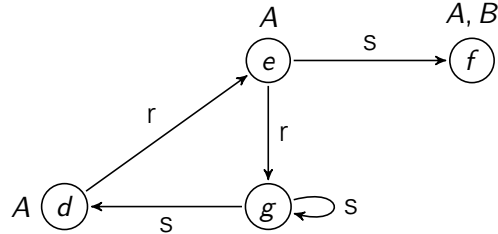
- $A \sqcup B$
- $\exists s. \neg A$
- $\forall s. A$
- $(\geq 2 s)$
- $\exists s. \exists s. \exists s. \exists s. A$
- $\forall s^{-1}. \exists s. \exists s. \exists s. A$
- $\neg \exists r. (\neg A \sqcap \neg B)$
- $\exists s. (A \sqcap \forall s. \neg B) \sqcap \neg \forall r. \exists r. (A \sqcup \neg A)$

Exercise 8

Consider the ABox

$$\mathcal{A} = \{A(d), A(e), A(f), B(f), r(d, e), r(e, g), s(e, f), s(g, g), s(g, d)\}$$

with the following graphical representation:



For each of the following \mathcal{ALC} -concepts C , list all individuals that are instances of C w.r.t. \mathcal{A} . Compare your results to Exercise 7.

- $A \sqcup B$
- $\exists s. \neg A$
- $\forall s. A$
- $\exists s. \exists s. \exists s. \exists s. A$
- $\neg \exists r. (\neg A \sqcap \neg B)$
- $\exists s. (A \sqcap \forall s. \neg B) \sqcap \neg \forall r. \exists r. (A \sqcup \neg A)$

Exercise 9

Consider the TBox

$$\mathcal{T} := \{\neg(A \sqcup B) \sqsubseteq \perp, A \sqsubseteq \neg B \sqcap \exists r. B, D \sqsubseteq \forall r. A, B \sqsubseteq \neg A \sqcap \exists r. A\},$$

the ABox

$$\mathcal{A} := \{r(a, b), r(a, c), r(a, d), r(d, c), (B \sqcap \forall r. D)(a), E(b), (\neg A)(c), (\exists s. \neg D)(d)\},$$

and the knowledge base $\mathcal{K} := \langle \mathcal{T}, \mathcal{A} \rangle$. Check for

- the TBox \mathcal{T} ,
- the ABox \mathcal{A} , and
- the knowledge base \mathcal{K}

whether it has a model. If it has one, specify such a model. If it does not have a model, explain why.