Description Logics

Exercise Sheet 3

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Exercise 10

Revisit the procedure for expanding TBoxes given in the proof of Proposition 2.6 of the lecture. Prove that

a) this procedure always terminates, and

b) that it returns a TBox that is equivalent to its input.

Hint for proving termination: Count, for each concept name A, the number of concept names (directly or indirectly) used in the definition of A.

Exercise 11

Prove that existential restrictions are monotonic, i.e. show that

$$C \sqsubseteq_{\mathcal{T}} D \quad \Longrightarrow \quad \exists r.C \sqsubseteq_{\mathcal{T}} \exists r.D.$$

Exercise 12

Prove the following result: Let $\mathcal{O} := \langle \mathcal{T}, \mathcal{A} \rangle$ be an ontology. If a is an instance of C w.r.t. $\mathcal{O}$ and $C \sqsubseteq_{\mathcal{T}} D$, then a is an instance of D w.r.t. $\mathcal{O}$.

Exercise 13

Prove the following results.

Let $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$ be an ontology, C an $\mathcal{ALC}$-concept, and a an individual name.

a) $\mathcal{O}$ is consistent iff $\tau(\mathcal{O})$ is consistent.

b) a is an instance of C w.r.t. $\mathcal{O}$ iff $\tau(\mathcal{O}) \models \tau_x(C)(a)$.

Exercise 14

Let $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$ be a consistent ontology. We write $C \sqsubseteq_{\mathcal{O}} D$ if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ for every model $\mathcal{I}$ of $\mathcal{O}$. Prove that for all $\mathcal{ALC}$-concepts C and D, we have $C \sqsubseteq_{\mathcal{O}} D$ iff $C \sqsubseteq_{\mathcal{T}} D$.

Hint: Use disjoint unions.