Description Logics

Exercise Sheet 4

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Winter Semester 2013/14

Exercise 15

In the lecture, we defined bisimulations for $\mathcal{ALC}$-concepts s.t. they capture the expressive power of $\mathcal{ALC}$, i.e. that bisimulation invariance for $\mathcal{ALC}$-concepts follows.

a) Extend the notion of bisimulation relation to $\mathcal{ALCN}$ s.t. bisimulation invariance for $\mathcal{ALCN}$-concepts follows.

b) Show bisimulation invariance for the bisimulation relation defined in exercise (a).

c) Prove that $\mathcal{ALCQ}$ is more expressive than $\mathcal{ALCN}$.

Exercise 16

Show the following claim:
If a concept C is satisfiable w.r.t. an $\mathcal{ALC}$-TBox $\mathcal{T}$, then for all $n \geq 1$ there is a model $\mathcal{I}_n$ of $\mathcal{T}$ such that: $|C^{\mathcal{I}_n}| \geq n$.

Exercise 17

Let $\rho_1 \subseteq \Delta^{\mathcal{I}_1} \times \Delta^{\mathcal{I}_2}$ and $\rho_2 \subseteq \Delta^{\mathcal{I}_2} \times \Delta^{\mathcal{I}_3}$ be bisimulation relations. Prove that bisimulations are closed under

a) composition (e.g. $\rho_3 = \rho_2 \circ \rho_1$ is a bisimulation), and

b) union (e.g. $\rho_3 = \rho_1 \cup \rho_2$ is a bisimulation).

Exercise 18

Prove or refute the following claim:
Given an $\mathcal{ALC}$-concept C and an $\mathcal{ALC}$-TBox $\mathcal{T}$. If $\mathcal{I}$ is an interpretation and $\mathcal{J}$ its filtration w.r.t. $\mathsf{sub}(C) \cup \mathsf{sub}(\mathcal{T})$, then the relation $\rho = \{(d, [d]) \mid d \in \Delta^{\mathcal{I}}, [d] \in \Delta^{\mathcal{J}},\ d \simeq [d]\}$ is a bisimulation.