



## Description Logics

### Exercise Sheet 5

Dr. rer. nat. Rafael Peñaloza / Marcel Lippmann  
Winter Semester 2013/14

#### Exercise 19

Let  $S$  be a finite set of concepts, and  $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$  be an interpretation. Prove or refute the following claim:

If  $S$  is closed, then  $t_S(d)$  is closed for every  $d \in \Delta^{\mathcal{I}}$ .

#### Exercise 20

Use the tableau algorithm from the lecture to decide whether the following subsumption holds:

$$\neg \forall r. A \sqcap \forall r. C \sqsubseteq_{\mathcal{T}} \forall r. E$$

where  $\mathcal{T} = \{ C \equiv (\exists r. \neg B) \sqcap \neg A, \quad D \equiv \exists r. B, \quad E \equiv \neg(\exists r. A) \sqcap \exists r. D \}$ .

#### Exercise 21

Consider the tableau algorithm from the lecture and extend it with the following two rules:

- *Condition:*  $\mathcal{A}$  contains  $(\geq n r)(a)$ , but  $k = |\{b \mid r(a, b) \in \mathcal{A}\}| < n$   
*Action:*  $\mathcal{A}' := \mathcal{A} \cup \{r(a, b_i) \mid k < i \leq n\}$  where  $b_i$  are new individual names
- *Condition:*  $\mathcal{A}$  contains  $(\leq n r)(a)$  and  $k = |\{b \mid r(a, b) \in \mathcal{A}\}| > n$   
*Action:*  $\mathcal{A}' := \mathcal{A} \cup \{A(b), \neg A(b)\}$  where  $A$  is a concept name and  $b$  is a new individual name

Is the obtained algorithm sound and complete for  $\mathcal{ALCN}$ ? Explain why.

#### Exercise 22

Extend the proof of Lemma 4.1 (local correctness) to the  $\sqcap$ -rule and the  $\forall$ -rule.