Description Logics

Exercise Sheet 8

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Exercise 31

Let $\mathcal{K} = \langle \mathcal{A}_0, \mathcal{T} \rangle$ be an $\mathcal{ALC}$-knowledge base, where $\mathcal{T}$ is a general TBox. The *precompletion of* $\mathcal{K}$ is the set of ABoxes M that is produced by the tableau algorithm when starting with the set of ABoxes $\{\mathcal{A}_0\}$ and exhaustively applying all tableau rules plus the GCI-rule except for the modified $\exists$-rule. Do the following:

a) Show that $\mathcal{K}$ is consistent iff there is an open ABox $\mathcal{A} \in M$ such that for all individual names a occurring in $\mathcal{A}$, the concept $C^a_{\mathcal{A}} := \bigsqcap_{C(a) \in \mathcal{A}} C$ is satisfiable w.r.t. $\mathcal{T}$.

   Hint: For the "if" direction, proceed as follows: The correctness of the tableau algorithm for $\mathcal{ALC}$ implies that, if $C^a_{\mathcal{A}}$ is satisfiable, then exhaustively applying all (!) rules to the set of ABoxes $\{\{C^a_{\mathcal{A}}(a)\}\}$ yields a set M' that contains an open and complete ABox. Show how to join all these ABoxes to obtain an open and complete tableau for $\mathcal{A}$ and conclude that $\mathcal{A}_0$ is consistent w.r.t. $\mathcal{T}$.

b) Use the result from a) to prove that ABox consistency in $\mathcal{ALC}$ can be decided in deterministic exponential time.

Exercise 32

For each of the following languages of binary trees over the alphabet $\Sigma = \{a, b\}$, define a looping tree automaton that accepts the language.

a) The set of all trees that contain a branch (starting at the root) in which all nodes are labelled with a.

b) The set of all trees that do not contain nodes n_0, n_1, n_2 such that

   - $n_1 = n_0 i$ for some $i \in \{0, 1\}$,
   - $n_2 = n_1 j$ for some $j \in \{0, 1\}$, and
   - $T(n_0) = T(n_1) = T(n_2) = a$.

Exercise 33

Show that there is no looping tree automaton on binary $\{a, b\}$-trees that accepts the set of all trees that contain a branch with infinitely many nodes labelled with a.