Description Logics

Exercise Sheet 11

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Exercise 40

Determine whether Player 2 has a winning strategy in the PSPACE game $G = (\phi, \{p_0, p_2\}, \{p_1, p_3\}, <)$ with

$$\phi = (\neg p_0 \rightarrow p_1) \wedge ((p_0 \wedge p_1) \rightarrow (p_2 \vee p_3)) \wedge (\neg p_1 \rightarrow (p_3 \rightarrow \neg p_2))$$

and $p_i < p_j$ iff $i < j$.

Exercise 41

A *quantified Boolean formula* is of the form $\phi = Q_1 p_1 \dots Q_n p_n.\psi$ where $Q_1, \dots, Q_n \in \{\forall, \exists\}$, $p_1, \dots, p_n$ are propositional variables, and ψ is propositional formula containing only the variables $p_1, \dots, p_n$.

Such a formula is *valid* iff the following is satisfied:

- For $n = 0$, the formula ϕ does not contain variables, and thus is a Boolean combination of 0 and 1. It is valid iff this Boolean combination of 0 and 1 evaluates to 1.
- For $n > 0$, we consider:

$$\phi_0 := Q_2 p_2 \dots Q_n p_n.\psi[p_1 := 0], \text{ and}$$
$$\phi_1 := Q_2 p_2 \dots Q_n p_n.\psi[p_1 := 1].$$

  If $Q_1 = \exists$, then ϕ is valid iff one of ϕ_0 and ϕ_1 is valid.
  If $Q_1 = \forall$, then ϕ is valid iff both ϕ_0 and ϕ_1 are valid.

QBF denotes the set of valid quantified Boolean formulae. Prove by reduction of QBF that the problem of deciding the existence of winning strategy for Player 2 in PSPACE games is PSPACE-hard.

Exercise 42

Finish the proof of Lemma 6.7 by showing that $v_0 \in C_G^{\mathcal{I}}$.

Exercise 43

Determine whether Player 2 has a winning strategy in the ExpTime game $G = (\phi, \Gamma_1, \Gamma_2, t_0)$ with

- $\phi = (p_1 \wedge p_2 \wedge p_3 \wedge \neg q) \vee (\neg p_1 \wedge \neg p_2 \wedge \neg p_3 \wedge q)$,
- $\Gamma_1 = \{p_1, p_2, p_3\}$,
- $\Gamma_2 = \{q\}$,
- $t_0(p_1) = t_0(p_2) = t_0(p_3) = t_0(q) = 0$.