TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Description Logics

Exercise Sheet 12

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Exercise 41

Let $f_1, \dots, f_m$ and $g_1, \dots, g_n$ be (not necessarily distinct) abstract features. A *feature agreement* is a concept of the form $(f_1 \circ \cdots \circ f_m) \downarrow (g_1 \circ \cdots \circ g_n)$ with the semantics:

$$\left((f_1 \circ \cdots \circ f_m) \downarrow (g_1 \circ \cdots \circ g_n)\right)^{\mathcal{I}} := \{d \in \Delta^{\mathcal{I}} \mid f_m^{\mathcal{I}}(\cdots f_2^{\mathcal{I}}(f_1^{\mathcal{I}}(d))\cdots) = g_n^{\mathcal{I}}(\cdots g_2^{\mathcal{I}}(g_1^{\mathcal{I}}(d))\cdots)\}$$

Feature disagreements ($\uparrow$) are defined analogously. The description logic $\mathcal{ALCF}$ extends $\mathcal{ALC}$ with feature agreements and feature disagreements.

Show that for $\mathcal{ALCF}$, satisfiability w.r.t. general TBoxes is undecidable.

Exercise 42

Let $\mathcal{D}$ be a concrete domain and $\mathcal{ALC}(\mathcal{D})$ denote the extension of $\mathcal{ALC}$ with the concrete domain $\mathcal{D}$. Show the following:

a) If f is an abstract feature, then $\exists f.C$ is equivalent to $\exists f.\top \sqcap \forall f.C$.

b) If $\mathcal{D}$ contains only unary predicates, every $\mathcal{ALC}(\mathcal{D})$ concept can be 'emulated' by a corresponding $\mathcal{ALCN}$ concept.

Exercise 43

A *role complement* is a role of the form $\neg r$, where r is a role name. The semantics of role complements is defined as follows:

$$(\neg r)^{\mathcal{I}} := \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \setminus r^{\mathcal{I}}.$$

The description logic $\mathcal{ALC}^{\neg}$ extends $\mathcal{ALC}$ by role complements, i.e. role complements are allowed to occur in existential restrictions, value restrictions and role assertions.

Show that $\mathcal{ALC}^{\neg}$ does not have the tree-model property.