Description Logics

Exercise Sheet 6

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Exercise 23

Prove Lemma 4.5 from the lecture.

Let $\mathcal{A} \in \mathcal{M}$ where $\mathcal{A}_0 \xrightarrow{*} \mathcal{M}$.

a) If $C(a) \in \mathcal{A}$, then $C \in \mathsf{Sub}(\mathcal{A}_0)$.

b) If $r(a, x) \in \mathcal{A}$ and x is a new individual, then $\max_{C(a) \in \mathcal{A}} |C| > \max_{C(x) \in \mathcal{A}} |C|$.

Hint: Use induction on the number of rule applications.

Exercise 24

Prove Lemma 4.6 from the lecture.

a) $|\mathsf{Sub}(\mathcal{A}_0)| \leq \widehat{m}$

b) $|C| \leq \widehat{m}$ for all $C \in \mathsf{Sub}(\mathcal{A}_0)$

c) If $\mathcal{A}$ is an ABox obtained by applying the tableau rules to $\mathcal{A}_0$ and x is an individual occuring in $\mathcal{A}$, then $\mathsf{dpt}_{\mathcal{A}}(x) \leq \widehat{m}$.

Exercise 25

Prove local correctness for the lazy expansion rule $\equiv_2$.

Exercise 26

The tableau algorithm for checking consistency of $\mathcal{ALC}$-ABoxes w.r.t. general TBoxes can be extended to inverse roles by adapting the $\exists$-rule and $\forall$-rule as follows:

Let C be an $\mathcal{ALCI}$-concept, and r an $\mathcal{ALCI}$-role, i.e. r denotes a role or an inverse role name, and $(r^{-1})^{-1} = r$ holds.

$\exists$-rule: Condition: $\mathcal{A}$ contains $(\exists r.C)(a)$, a is not blocked, but there is no b with either $\{r(a, b), C(b)\} \subseteq \mathcal{A}$ or $\{r^{-1}(b, a), C(b)\} \subseteq \mathcal{A}$

Action: $\mathcal{A}' := \mathcal{A} \cup \{r(a, b), C(b)\}$ for a new individual b not occuring in $\mathcal{A}$

$\forall$-rule: Condition: $(\forall r.C)(a) \in \mathcal{A}$ and $r(a, b) \in \mathcal{A}$ or $r^{-1}(b, a) \in \mathcal{A}$, but $C(b) \notin \mathcal{A}$

Action: $\mathcal{A}' := \mathcal{A} \cup \{C(b)\}$

a) Does it suffice to use subset blocking to obtain a decision procedure?

b) Is the extended tableau algorithm for $\mathcal{ALCI}$ sound and complete?

Exercise 27

Compare the tableau algorithm (with eager expansion) to the tableau algorithm extended with the $\equiv_1$- and $\equiv_2$-rules (lazy expansion) by applying both methods to check whether A is satisfiable w.r.t. $\mathcal{T}$, where

$$\mathcal{T} := \{A \equiv \neg B \sqcap B,\ B \equiv \exists r.\exists s.(C \sqcap D)\}.$$

a) What is the maximal number of complete ABoxes obtained in the set of ABoxes by eager expansion? What is the minimal number for lazy expansion?

b) What is the maximal number of rule applications by eager expansion? What is the minimal number for lazy expansion?

c) What is the maximal number of assertions in a complete ABox obtained by eager expansion? What is the minimal number for lazy expansion?

d) Give $\kappa(\mathcal{M})$ for all sets $\mathcal{M}$ of ABoxes considered in the tableau algorithm with eager expansion.

e) Give $\kappa_{\mathcal{T}}(\mathcal{M})$ for all sets $\mathcal{M}$ of ABoxes considered in the tableau algorithm with lazy expansion.