TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Description Logics

Exercise Sheet 8

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Exercise 31

For each of the following languages of binary trees over the alphabet $\Sigma = \{a, b\}$, define a looping tree automaton that accepts it.

a) The set of all trees that contain a branch (starting at the root) in which all nodes are labelled with a.

b) The set of all trees that do not contain nodes n_0, n_1, n_2 such that

- $n_1 = n_0 i$ for some $i \in \{0, 1\}$,
- $n_2 = n_1 j$ for some $j \in \{0, 1\}$, and
- $T(n_0) = T(n_1) = T(n_2) = a$.

Exercise 32

Show that there is no looping tree automaton on binary $\{a, b\}$-trees that accepts the set of all trees that contain a branch with infinitely many nodes labelled with a.

Exercise 33

Reconsider the claim: for all $D \in S_{C,\mathcal{T}}$, we have $D \in R(u) \implies u \in D^{\mathcal{I}_R}$. Show the claim by induction on the structure of D for the missing cases:

- $D = D_1 \sqcup D_2$ and
- $D = \forall r.E$.

Exercise 34

Recall the following: A *propositional Horn clause* is of the form $p_1, \ldots, p_k \to p$ where $p_1, \ldots, p_k$ are propositional variables and p is a propositional variable or $\perp$. A *propositional Horn formula* is a finite set of propositional Horn clauses. The satisfiability problem of propositional Horn formulas can be decided in linear time.

Show that the emptiness problem for looping tree automata can be decided in linear time by giving a linear-time reduction to the satisfiability problem of propositional Horn formulas.