TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Description Logics

Exercise Sheet 10

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Exercise 38

Consider the $\mathcal{EL}$-TBox $\mathcal{T}$ consisting of the following axioms:

$$\begin{aligned} A &\sqsubseteq B \sqcap \exists r.C \\ B \sqcap \exists r.B &\sqsubseteq C \sqcap D \\ C &\sqsubseteq \exists r.A \sqcap B \\ \exists r.\exists r.B \sqcap D &\sqsubseteq \exists r.(A \sqcap B) \end{aligned}$$

Check whether the following subsumption relations hold w.r.t. $\mathcal{T}$:

a) $A \sqsubseteq B$

b) $A \sqsubseteq \exists r.\exists r.A$

c) $B \sqcap \exists r.A \sqsubseteq \exists r.C$

Exercise 39

Determine whether Player 2 has a winning strategy in the PSPACE game
$G = (\phi, \{p_0, p_2\}, \{p_1, p_3\}, <)$ with

$$\phi = (\neg p_0 \to p_1) \land ((p_0 \land p_1) \to (p_2 \lor p_3)) \land (\neg p_1 \to (p_3 \to \neg p_2))$$

and $p_i < p_j$ iff $i < j$.

Exercise 40

A *quantified Boolean formula* is of the form $\phi = Q_1 p_1 \ldots Q_n p_n.\psi$ where $Q_1, \ldots, Q_n \in \{\forall, \exists\}$ are quantifiers, $p_1, \ldots, p_n$ are propositional variables, and ψ is a propositional formula containing only the variables $p_1, \ldots, p_n$.

Validity of such formulae is defined as follows:

- For $n = 0$, the formula ϕ does not contain variables, and thus is a Boolean combination of 0 and 1. Then ϕ is valid iff it evaluates to 1.
- For $n > 0$, we consider:

$$\begin{aligned} \phi_0 &:= Q_2 p_2. \ldots Q_n p_n.\psi[p_1 := 0], \text{ and} \\ \phi_1 &:= Q_2 p_2. \ldots Q_n p_n.\psi[p_1 := 1]. \end{aligned}$$

  If $Q_1 = \exists$, then ϕ is valid iff one of ϕ_0 and ϕ_1 is valid.
  If $Q_1 = \forall$, then ϕ is valid iff both ϕ_0 and ϕ_1 are valid.

QBF denotes the set of all valid quantified Boolean formulae. Prove by reduction of QBF that the problem of deciding the existence of winning strategy for Player 2 in PSPACE games is PSPACE-hard.

Exercise 41

Finish the proof of Lemma 6.7 by showing that $v_0 \in C_G^{\mathcal{I}}$.