Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Description Logics

Exercise Sheet 13

Dr.-Ing. habil. Anni-Yasmin Turhan / Francesco Kriegel
Winter Semester 2014/15

Exercise 49

Consider the concrete domain $\mathcal{N} := (\Delta^{\mathcal{N}}, \Phi^{\mathcal{N}})$ defined in section 7.2 of the lecture. Show that $\mathcal{N}$ is admissible.

$$\begin{aligned} \Delta^{\mathcal{N}} &:= \mathbb{N} \\ \Phi^{\mathcal{N}} &:= \{=, =_0, +1, \top_{\mathcal{N}}\} \cup \{\text{negations of } =, =_0, +1, \top_{\mathcal{N}}\} \end{aligned}$$

Exercise 50

Prove that for a given TBox $\mathcal{T}$ and a consequence $A \sqsubseteq B$ there can be exponentially many MinAs for $A \sqsubseteq B$.

Hint: You do not need any concept constructors except conjunction.

Exercise 51

Let $\mathcal{L}_1$ and $\mathcal{L}_2$ be two DL-languages. We define the $\mathcal{L}_2$-lcs as follows. Let $C_1, \dots, C_n$ be $\mathcal{L}_1$-concept descriptions. An $\mathcal{L}_2$-concept description C is called the *$\mathcal{L}_2$-lcs of* $C_1, \dots, C_n$ iff

(1) $C_i \sqsubseteq C$ holds for all i, $1 \leq i \leq n$, and

(2) for all $\mathcal{L}_2$-concept descriptions D it holds that $C_i \sqsubseteq D$ for all i, $1 \leq i \leq n$ imply $C \sqsubseteq D$.

This generalizes the standard notion of least common subsumers since it allows different logics for $C_1, \dots, C_n$ and their lcs.

Consider the following $\mathcal{FLE}$-concept descriptions

$$\begin{aligned} C_1 &= \exists r.C \sqcap \exists r.D \sqcap \forall r.(A \sqcap B), \\ C_2 &= \exists r.B \sqcap \exists r.D \sqcap \forall r.(A \sqcap D). \end{aligned}$$

Find

a) the $\mathcal{EL}$-lcs,

b) the $\mathcal{FLE}$-lcs[1], and

c) the $\mathcal{ALC}$-lcs

of C_1 and C_2.

Hint: It is not necessary to provide an algorithm that can compute the lcs in the general case.

[1] $\mathcal{FLE}$ provides conjunction, existential restrictions and value restrictions.

Exercise 52

Let $\mathcal{K} = (\mathcal{A}, \mathcal{T})$ be a knowledge base and $i_1, \dots, i_n$ individuals occuring in $\mathcal{A}$.

A concept description C is the *most specific concept of* $i_1, \dots, i_n$ *w.r.t.* $\mathcal{K}$, iff

(1) all individuals $i_1, \dots, i_n$ are instances of C w.r.t. $\mathcal{K}$, and

(2) for all concept descriptions D it holds that, whenever all individuals $i_1, \dots, i_n$ are instances of D w.r.t. $\mathcal{K}$, then $D \sqsubseteq_{\mathcal{T}} C$ holds.

Prove, that the most specific concept of $i_1, \dots, i_n$ w.r.t. $\mathcal{K}$ is equivalent to the least common subsumer of the most specific concepts of $i_1, \dots, i_n$ w.r.t. $\mathcal{K}$.