TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Introduction to Complexity Theory

Exercise Sheet 2

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Winter Semester 2014/15

Exercise 4

Determine which of the following statements are correct. Justify your answers.

a) $2n \in O(n)$

b) $2n \in o(n)$

c) $1 \in o(n)$

d) $n \in O(n \cdot \log n)$

e) $n \cdot \log n \in o(n^2)$

f) $2^{2^n} \in O(2^n)$

g) $2^{2n} \in o(2^n)$

h) $4^n \in O(2^n)$

i) $4^n \in 2^{O(n)}$

Exercise 5

Show that for $f, g \colon \mathbb{N} \to \mathbb{N}_{>0}$, we have:

$$f(n) \in o(g(n)) \implies f(n) \in O(g(n)) \text{ and } g(n) \notin O(f(n)).$$

Moreover, show that the following two functions are a counterexample for the converse direction:

- $f(n) = 0.5 \cdot n$;
- $g(n) = \begin{cases} 2^n & \text{if } n \text{ is even} \\ n & \text{if } n \text{ is odd} \end{cases}$

Exercise 6

Let $f, g \colon \mathbb{N} \to \mathbb{N}$. We write $f(n) \in \Theta(g(n))$ if $f(n) \in O(g(n))$ and $g(n) \in O(f(n))$. Show that if $p(n)$ is a polynomial of degree d, then $p(n) \in \Theta(n^d)$.

Exercise 7

Devise a non-deterministic Turing machine that accepts the following language L over $\Sigma := \{a, b\}$. For each $w_1 w_2 \dots w_n \in L$, there is a position i, $1 \leq i \leq n-2$, such that $w_i = a$ and $w_{i+2} \neq b$.

Exercise 8

Finish the proof of Theorem 2.4 of the lecture by showing that

$$(d^{T(n)+1} - 1) \cdot c \cdot T(n) \in 2^{O(T(n))}.$$

Exercise 9

Show that

a) the functions $f_1(n) = 2n$ and $f_2(n) = n^2$ are time-constructible; and

b) the functions $g_1(n) = 2^n$ and $g_2(n) = n!$ is space-constructible.

An informal proof sketch is sufficient.