Introduction to Complexity Theory

Exercise Sheet 3

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Exercise 9

Prove the following: If $f, g\colon \mathbb{N} \to \mathbb{N}$ are space-constructible, then so are $f + g$, $f \cdot g$, and 2^f.

Exercise 10

Prove the *space-compression theorem* (Theorem 4.2 of the lecture): For all $\varepsilon \in (0, 1]$, and all $S\colon \mathbb{N} \to \mathbb{N}$, we have $\mathsf{DSpace}(S) \subseteq \mathsf{DSpace}(\max\{n, \lceil \varepsilon \cdot S(n) \rceil\})$.

Exercise 11

In the lecture, Turing machines were used to decide languages. DTMs can, however, also be used to compute functions, where we assume that the argument is given on the input tape (in unary coding), and the Turing machine stops after having written the result on the output tape. A function $f\colon \mathbb{N} \to \mathbb{N}$ is called *computable* if there is a DTM that computes f.

Show that $h(n) := \lfloor \frac{n}{2} \rfloor$ is computable.

Exercise 12

Prove the *gap theorem for time*: For every total computable function $g\colon \mathbb{N} \to \mathbb{N}$ with $g(n) \geq n$, there is a total computable function $T\colon \mathbb{N} \to \mathbb{N}$ such that $\mathsf{DTime}(T) = \mathsf{DTime}(g \circ T)$.