TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Introduction to Complexity Theory

Exercise Sheet 5

Dr. Rafael Peñaloza / Dr. Marcel Lippmann

Winter Semester 2014/15

Exercise 17

Prove that the following problem can be solved in polynomial time: Given a propositional formula ϕ using variables $p_1, \dots, p_k$ and a truth assignment $w \in \{0, 1\}^k$, check whether w is a model of ϕ.

Exercise 18

Show that the following problem is in PSPACE. Given a non-deterministic finite automaton $\mathcal{A}$ over an alphabet Σ, is $L(\mathcal{A}) = \Sigma^*$.

Hint: Prove that, if $L(\mathcal{A}) \neq \Sigma^*$ and $\mathcal{A}$ has n states, then there exists a word $w \in \Sigma^*$ of length at most 2^n such that $w \notin L(\mathcal{A})$. Then, use this fact to give a non-deterministic algorithm whose space consumption is polynomially bounded. Finally, apply Savitch's Theorem.

Exercise 19

A *quantified Boolean formula* is of the form $\phi = Q_1 p_1 \dots Q_n p_n.\psi$ where $Q_1, \dots, Q_n \in \{\forall, \exists\}$, $p_1, \dots, p_n$ are propositional variables, and ψ is propositional formula containing only the variables $p_1, \dots, p_n$.

Such a formula is *valid* iff the following is satisfied:

- For $n = 0$, the formula ϕ does not contain variables, and thus is a Boolean combination of 0 and 1. It is valid iff this Boolean combination of 0 and 1 evaluates to 1.
- For $n > 0$, we consider:

$$\phi_0 := Q_2 p_2 \dots Q_n p_n.\psi[p_1 := 0], \text{ and}$$
$$\phi_1 := Q_2 p_2 \dots Q_n p_n.\psi[p_1 := 1].$$

  If $Q_1 = \exists$, then ϕ is valid iff one of ϕ_0 and ϕ_1 is valid.
  If $Q_1 = \forall$, then ϕ is valid iff both ϕ_0 and ϕ_1 are valid.

QBF denotes the set of valid quantified Boolean formulae. Prove that QBF $\in$ PSPACE.