



## Introduction to Complexity Theory

### Exercise Sheet 6

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#### Exercise 20

For each of the following statements, say whether it is true or false. Justify your answers.

- a)  $3SAT \in PSPACE$ ;
- b)  $SAT \leq_p CLIQUE$ ;
- c)  $CLIQUE \leq_p SAT$ ;
- d)  $LOGSPACE \neq EXPTIME$ ; and
- e) if  $L \leq_p L'$  and  $L'$  is NP-hard, then  $L$  is NP-hard.

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#### Exercise 21

HALF-CLIQUE is the problem to decide, given a graph  $G$  with  $n \geq 1$  nodes, whether  $G$  contains a clique of size  $\lceil n/2 \rceil$ . Prove that HALF-CLIQUE is NP-complete.

#### Exercise 22

Prove Lemma 4.14 from the lecture: Let  $\mathcal{C}$  be a complexity class closed under polynomial-time reductions. If  $L$  is  $\mathcal{C}$ -hard and  $L \leq_p L'$ , then  $L'$  is  $\mathcal{C}$ -hard.

#### Exercise 23

Complete the proof of Theorem 4.13 of the lecture by showing that  $M$  accepts  $w$  iff  $\phi_w := \phi_{ini} \wedge \phi_{move} \wedge \phi_{keep} \wedge \phi_{acc} \wedge \phi_{aux}$  is satisfiable.