TECHNISCHE
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Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Description Logics

Exercise Sheet 3

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Exercise 10

Consider the TBox

$$\mathcal{T} := \{\neg(A \sqcup B) \sqsubseteq \bot, \quad A \sqsubseteq \neg B \sqcap \exists r.B, \quad D \sqsubseteq \forall r.A, \quad B \sqsubseteq \neg A \sqcap \exists r.A\},$$

the ABox

$$\mathcal{A} := \{r(a,b),\ r(a,c),\ r(a,d),\ r(d,c),\ (B \sqcap \forall r.D)(a),\ E(b),\ (\neg A)(c),\ (\exists s.\neg D)(d)\},$$

and the knowledge base $\mathcal{K} := \langle \mathcal{T}, \mathcal{A} \rangle$. Check for

a) the TBox $\mathcal{T}$,

b) the ABox $\mathcal{A}$, and

c) the knowledge base $\mathcal{K}$,

whether it has a model. If it has one, specify such a model. If it does not have a model, explain why.

Exercise 11

Prove that existential restrictions are monotonic, i.e. show that

$$C \sqsubseteq_{\mathcal{T}} D \quad \Longrightarrow \quad \exists r.C \sqsubseteq_{\mathcal{T}} \exists r.D.$$

Exercise 12

Prove the following result: Let $\mathcal{K} := \langle \mathcal{T}, \mathcal{A} \rangle$ be a knowledge base. If a is an instance of C w.r.t. $\mathcal{K}$ and $C \sqsubseteq_{\mathcal{T}} D$, then a is an instance of D w.r.t. $\mathcal{K}$.

Exercise 13

Prove the following results. Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ be a knowledge base, C an $\mathcal{ALC}$-concept, and a an individual name.

a) $\mathcal{K}$ is consistent iff $\tau(\mathcal{K})$ is consistent.

b) a is an instance of C w.r.t. $\mathcal{K}$ iff $\tau(\mathcal{K}) \models \tau_x(C)(a)$.

Exercise 14

Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ be a consistent knowledge base. We write $C \sqsubseteq_{\mathcal{K}} D$ if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ for every model $\mathcal{I}$ of $\mathcal{K}$. Prove that for all $\mathcal{ALC}$-concepts C and D, we have $C \sqsubseteq_{\mathcal{K}} D$ iff $C \sqsubseteq_{\mathcal{T}} D$.

Hint: Use disjoint unions.