Description Logics

Exercise Sheet 4

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Exercise 15

In the lecture, we defined bisimulations for \( \mathcal{ALC} \)-concepts s.t. they capture the expressive power of \( \mathcal{ALC} \), i.e. that bisimulation invariance for \( \mathcal{ALC} \)-concepts follows.

\begin{itemize}
  \item[a)] Extend the notion of a bisimulation relation to \( \mathcal{ALCN} \) s.t. bisimulation invariance for \( \mathcal{ALCN} \)-concepts follows.
  \item[b)] Show bisimulation invariance for the bisimulation relation defined in exercise (a).
  \item[c)] Prove that \( \mathcal{ALCQ} \) is more expressive than \( \mathcal{ALCN} \).
\end{itemize}

Exercise 16

Prove or refute the following claim:

If an \( \mathcal{ALC} \)-concept \( C \) is satisfiable w.r.t. an \( \mathcal{ALC} \)-TBox \( T \), then for all \( n \geq 1 \) there is a model \( I_n \) of \( T \) such that: \( |C^{I_n}| \geq n \).

Exercise 17

Prove that bisimulations are closed under

\begin{itemize}
  \item[a)] composition \( \circ \), and
  \item[b)] union \( \cup \).
\end{itemize}

Exercise 18

Prove or refute the following claim:

Given an \( \mathcal{ALC} \)-concept \( C \) and an \( \mathcal{ALC} \)-TBox \( T \). If \( I \) is an interpretation and \( J \) its filtration w.r.t. \( \text{sub}(C) \cup \text{sub}(T) \), then the relation \( \rho = \{ (d, [d]) \mid d \in \Delta^I \} \) is a bisimulation.