TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Description Logics

Exercise Sheet 5

PD Dr.-Ing. habil. Anni-Yasmin Turhan / Dipl.-Math. Francesco Kriegel
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Exercise 19

Let S be a finite set of concepts, and $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ be an interpretation. Prove or refute the following claim:
If S is closed, then $t_S(d)$ is closed for every $d \in \Delta^{\mathcal{I}}$.

Exercise 20

Use the tableau algorithm from the lecture to decide whether the following subsumption holds:

$$\neg\forall r.A \sqcap \forall r.C \sqsubseteq_{\mathcal{T}} \forall r.E$$

where $\mathcal{T} = \{C \equiv (\exists r.\neg B) \sqcap \neg A, \quad D \equiv \exists r.B, \quad E \equiv \neg(\exists r.A) \sqcap \exists r.D\}$.

Exercise 21

Extend the proof of Lemma 4.1 (local correctness) to the $\sqcap$-rule and the $\forall$-rule.

Exercise 22

Consider the tableau algorithm from the lecture and extend it with the following two rules:

- *Condition:* $\mathcal{A}$ contains $(\geq n\ r)(a)$, but $k = |\{b \mid r(a,b) \in \mathcal{A}\}| < n$
  Action: $\mathcal{A}' := \mathcal{A} \cup \{r(a, b_i) \mid k < i \leq n\}$ where b_i are new individual names
- *Condition:* $\mathcal{A}$ contains $(\leq n\ r)(a)$ and $k = |\{b \mid r(a,b) \in \mathcal{A}\}| > n$
  Action: $\mathcal{A}' := \mathcal{A} \cup \{A(b), \neg A(b)\}$ where A is a concept name and b is a new individual name

Is the obtained algorithm sound and complete for $\mathcal{ALCN}$? Explain why.