



Description Logics

Exercise Sheet 6

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Exercise 23

Prove Lemma 4.5 from the lecture.

Let $\mathcal{A} \in \mathcal{M}$ where $\mathcal{A}_0 \xrightarrow{*} \mathcal{A}$.

- If $C(a) \in \mathcal{A}$, then $C \in \text{Sub}(\mathcal{A}_0)$.
- If $r(a, x) \in \mathcal{A}$ and x is a new individual, then $\max_{C(a) \in \mathcal{A}} |C| > \max_{C(x) \in \mathcal{A}} |C|$.

Hint: Use induction on the number of rule applications.

Exercise 24

Prove Lemma 4.6 from the lecture.

- $|\text{Sub}(\mathcal{A}_0)| \leq \hat{m}$
- $|C| \leq \hat{m}$ for all $C \in \text{Sub}(\mathcal{A}_0)$
- If \mathcal{A} is an ABox obtained by applying the tableau rules to \mathcal{A}_0 and x is an individual occurring in \mathcal{A} , then $\text{dpt}_{\mathcal{A}}(x) \leq \hat{m}$.

Exercise 25

Prove local correctness for the lazy expansion rule \equiv_2 .

Exercise 26

Compare the tableau algorithm (with eager expansion) to the tableau algorithm extended with the \equiv_1 - and \equiv_2 -rules (lazy expansion) by applying both methods to check whether A is satisfiable w.r.t. \mathcal{T} , where

$$\mathcal{T} := \{A \equiv \neg B \sqcap B, B \equiv \exists r. \exists s. (C \sqcap D)\}.$$

- What is the maximal number of complete ABoxes obtained in the set of ABoxes by eager expansion? What is the minimal number for lazy expansion?
- What is the maximal number of rule applications by eager expansion? What is the minimal number for lazy expansion?
- What is the maximal number of assertions in a complete ABox obtained by eager expansion? What is the minimal number for lazy expansion?

- d) Give $\kappa(\mathcal{M})$ for all sets \mathcal{M} of ABoxes considered in the tableau algorithm with eager expansion.
- e) Give $\kappa_{\mathcal{T}}(\mathcal{M})$ for all sets \mathcal{M} of ABoxes considered in the tableau algorithm with lazy expansion.