Description Logics

Exercise Sheet 7

PD Dr.-Ing. habil. Anni-Yasmin Turhan / Dipl.-Math. Francesco Kriegel
Winter Semester 2015/16

Exercise 27

The tableau algorithm for checking consistency of $\mathcal{ALC}$-ABoxes w.r.t. general TBoxes can be extended to inverse roles by adapting the $\exists$-rule and $\forall$-rule as follows:

Let C be an $\mathcal{ALCI}$-concept, and r an $\mathcal{ALCI}$-role, i.e. r denotes a role or an inverse role name, and $(r^{-1})^{-1} = r$ holds.

$\exists$-rule: Condition: $\mathcal{A}$ contains $(\exists r.C)(a)$, a is not blocked, but there is no b with either $\{r(a,b), C(b)\} \subseteq \mathcal{A}$ or $\{r^{-1}(b,a), C(b)\} \subseteq \mathcal{A}$

Action: $\mathcal{A}' := \mathcal{A} \cup \{r(a,b), C(b)\}$ for a new individual b not occuring in $\mathcal{A}$

$\forall$-rule: Condition: $(\forall r.C)(a) \in \mathcal{A}$ and $r(a,b) \in \mathcal{A}$ or $r^{-1}(b,a) \in \mathcal{A}$, but $C(b) \notin \mathcal{A}$

Action: $\mathcal{A}' := \mathcal{A} \cup \{C(b)\}$

a) Does it suffice to use subset blocking to obtain a decision procedure?

b) Is the extended tableau algorithm for $\mathcal{ALCI}$ sound and complete?

Exercise 28

We consider another form of blocking, where an individual can be blocked by an individual that is not necessarily an ancestor: *anywhere blocking*. Instead of the depth of an individual and the ancestor relation, it uses the age of an individual and the relation $<$.

The *age* of an individual x, denoted by $\mathsf{age}(x)$, is defined as 0 for old individuals and n for a new individual x, that was generated by the nth application of the $\exists$-rule.

Let $\mathcal{A}$ be an ABox obtained by applying the tableau rules and the $\sqsubseteq$-rule to an initial ABox $\mathcal{A}_0$. A new individual x is *anywhere blocked* by an individual a in $\mathcal{A}$ iff

- $\{C \mid C(x) \in \mathcal{A}\} \subseteq \{D \mid D(a) \in \mathcal{A}\}$, and
- $\mathsf{age}(a) < \mathsf{age}(x)$.

Prove that the tableau algorithm with anywhere blocking is a decision procedure for consistency of $\mathcal{ALC}$-knowledge bases with general TBoxes.

Hint: For what subset of the complete tableau do we need to construct a model?

Exercise 29

Let $\mathcal{K} = \langle \mathcal{A}_0, \mathcal{T} \rangle$ be an $\mathcal{ALC}$-knowledge base, where $\mathcal{T}$ is a general TBox. The *precompletion of* $\mathcal{K}$ is the set of ABoxes $\mathcal{M}$ that is produced by the tableau algorithm when starting with the set of ABoxes $\{\mathcal{A}_0\}$ and exhaustively applying all tableau rules except the modified $\exists$-rule.

a) Show that $\mathcal{K}$ is consistent iff there is an open ABox $\mathcal{A} \in \mathcal{M}$ such that for all individual names a occurring in $\mathcal{A}$, the concept $C^a_{\mathcal{A}} := \bigsqcap_{C(a) \in \mathcal{A}} C$ is satisfiable w.r.t. $\mathcal{T}$.

   Hint: For the "if" direction, proceed as follows: The correctness of the tableau algorithm for $\mathcal{ALC}$ implies that, if $C^a_{\mathcal{A}}$ is satisfiable, then exhaustively applying all (!) rules to the set of ABoxes $\{\{C^a_{\mathcal{A}}(a)\}\}$ yields a set $\mathcal{M}'$ that contains an open and complete ABox. Show how to join all these ABoxes to obtain an open and complete tableau for $\mathcal{A}$ and conclude that $\mathcal{A}_0$ is consistent w.r.t. $\mathcal{T}$.

b) Use the result from a) to prove that ABox consistency in $\mathcal{ALC}$ can be decided in deterministic exponential time.