Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Description Logics

Exercise Sheet 10

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Exercise 37

Let $\mathcal{T}$ be a general $\mathcal{EL}$-TBox and $\widehat{\mathcal{T}}$ the TBox obtained from exhaustive application of the normalisation rules NF1–NF5 to $\mathcal{T}$.

Finish the proof of Lemma 5.18 by showing the following:

a) $\widehat{\mathcal{T}}$ can be obtained from $\mathcal{T}$ in polynomial time.

b) $\widehat{\mathcal{T}}$ is in normal form.

c) For all concept names A, B occurring in $\mathcal{T}$, we have $A \sqsubseteq_{\mathcal{T}} B$ iff $A \sqsubseteq_{\widehat{\mathcal{T}}} B$.

Exercise 38

Consider the $\mathcal{EL}$-TBox $\mathcal{T}$ consisting of the following axioms:

$$\begin{aligned} A &\sqsubseteq B \sqcap \exists r.C \\ B \sqcap \exists r.B &\sqsubseteq C \sqcap D \\ C &\sqsubseteq \exists r.A \sqcap B \\ \exists r.\exists r.B \sqcap D &\sqsubseteq \exists r.(A \sqcap B) \end{aligned}$$

Check whether the following subsumption relations hold w.r.t. $\mathcal{T}$:

a) $A \sqsubseteq B$

b) $A \sqsubseteq \exists r.\exists r.A$

c) $B \sqcap \exists r.A \sqsubseteq \exists r.C$

Exercise 39

A *role complement* is a role of the form $\neg r$, where r is a role name. The semantics of role complements is defined as follows:

$$(\neg r)^{\mathcal{I}} := \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \setminus r^{\mathcal{I}}.$$

The description logic $\mathcal{ALC}^{\neg}$ extends $\mathcal{ALC}$ by role complements, i.e. role complements are allowed to occur in existential restrictions, value restrictions and role assertions.

Show that $\mathcal{ALC}^{\neg}$ does not have the tree-model property.

Exercise 40

Use a tableau algorithm to decide whether the following $\mathcal{ALC}$-knowledge base is consistent:

$$\mathcal{T} := \{A \sqcap \forall r.\neg A \sqsubseteq \bot\}$$
$$\mathcal{A} := \{(\forall r.\neg A)(a),\ (\exists r.A)(b),\ r(a,b)\}$$

Exercise 41

For each of the following $\mathcal{ALC}$-concept descriptions C and $\mathcal{ALC}$-TBoxes $\mathcal{T}$ decide whether C is satisfiable w.r.t. $\mathcal{T}$ by constructing the looping tree automaton $\mathcal{A}_{C,\mathcal{T}}$ and checking its accepted language $L(\mathcal{A}_{C,\mathcal{T}})$ for emptiness.

a) $C := A$
$\mathcal{T} := \{A \sqsubseteq \neg A\}$

b) $C := A$
$\mathcal{T} := \emptyset$

c) $C := A \sqcap \exists r.A$
$\mathcal{T} := \{A \sqsubseteq \forall r.\neg A\}$