Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Description Logics

Exercise Sheet 11

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Exercise 42

Determine whether Player 2 has a winning strategy in the PSPACE game $G = (\phi, \{p_0, p_2\}, \{p_1, p_3\}, <)$ with

$$\phi = (\neg p_0 \to p_1) \wedge ((p_0 \wedge p_1) \to (p_2 \vee p_3)) \wedge (\neg p_1 \to (p_3 \to \neg p_2))$$

and $p_i < p_j$ iff $i < j$.

Exercise 43

A *quantified Boolean formula* is of the form $\phi = Q_1 p_1 \ldots Q_n p_n.\psi$ where $Q_1, \ldots, Q_n \in \{\forall, \exists\}$ are quantifiers, $p_1, \ldots, p_n$ are propositional variables, and ψ is a propositional formula containing only the variables $p_1, \ldots, p_n$.

Validity of such formulae is defined as follows:

- For $n = 0$, the formula ϕ does not contain variables, and thus is a Boolean combination of 0 and 1. Then ϕ is valid iff it evaluates to 1.
- For $n > 0$, we consider:

  $$\phi_0 := Q_2 p_2. \ldots Q_n p_n.\psi[p_1 := 0], \text{ and}$$
  $$\phi_1 := Q_2 p_2. \ldots Q_n p_n.\psi[p_1 := 1].$$

  If $Q_1 = \exists$, then ϕ is valid iff one of ϕ_0 and ϕ_1 is valid.
  If $Q_1 = \forall$, then ϕ is valid iff both ϕ_0 and ϕ_1 are valid.

QBF denotes the set of all valid quantified Boolean formulae. Prove that the problem of deciding the existence of winning strategy for Player 2 in PSPACE games is PSPACE-complete.

Exercise 44

Determine whether Player 2 has a winning strategy in the EXPTIME game $G = (\phi, \Gamma_1, \Gamma_2, t_0)$ with

- $\phi = (p_1 \wedge p_2 \wedge p_3 \wedge \neg q) \vee (\neg p_1 \wedge \neg p_2 \wedge \neg p_3 \wedge q)$,
- $\Gamma_1 = \{p_1, p_2, p_3\}$,
- $\Gamma_2 = \{q\}$,
- $t_0(p_1) = t_0(p_2) = t_0(p_3) = t_0(q) = 0$.